A.2 AASHTO Type IV, LRFD Specifications

A.2.1 INTRODUCTION

Detailed example showing sample calculations for design of typical Interior AASHTO Type–IV prestressed concrete Beam supporting single span bridge. The design is based on AASHTO LRFD Bridge Design Specifications 3rd Edition 2004.

A.2.2 DESIGN PARAMETERS

The bridge considered for design has a span length of 110 ft. (c/c pier distance) with no skew and a total width of 46 ft. The bridge superstructure consists of 6 AASHTO Type IV beams spaced 8 ft. center to center designed to act compositely with 8 in. thick cast in place concrete deck as shown in figure A.2.2.1. The wearing surface thickness is 1.5 in. which includes the thickness of any future wearing surface. T501 type rails are considered in the design. HL-93 is the design live load. The relative humidity of 60% is considered in the design. The bridge cross section is shown in fig A.2.2.1.

![Figure A.2.2.1 Bridge Cross Section](image)

A.2.3 MATERIAL PROPERTIES

Cast in place slab: Thickness $t_s = 8.0$ in.

Concrete Strength at 28-days, $f'_c = 4,000$ psi

Thickness of asphalt wearing surface (including any future wearing surfaces), $t_w = 1.5$ in.

Unit weight of concrete = 150 pcf

Precast beams: AASHTO Type-IV

Concrete Strength at release, $f'_{ci} = 4000$ psi (This value is taken as initial guess and will be finalized based on most optimum design)

Concrete Strength at 28 days, $f'_c = 5000$ psi (This value is taken as initial guess and will be finalized based on most optimum design)

Concrete unit weight = 150 pcf
Fig A.2.3.1 Beam end Details
(Adapted from TxDOT Standard drawing ibebstel)

Span Length (c/c Piers) = 110'-0"
From fig. A.2.3.1
Overall beam length = 110' – 2(2") = 109'-8"
Design Span = 110' – 2(8.5") = 108'-7" = 108.583’(c/c of bearing)

Pretensioning Strands: ½ in. diameter, seven wire low relaxation
Area of one strand = 0.153 in.²
Ultimate Stress, f_{pu} = 270,000 psi
Yield Strength, f_{py} = 0.9f_{pu} = 243,000 psi [LRFD Table 5.4.4.1-1]
Stress limits for prestressing strands:
before transfer, f_{pi} ≤ 0.75 f_{pu} = 202,500 psi
at service limit state(after all losses) f_{pe} ≤ 0.80 f_{py} = 194,400 psi
Modulus of Elasticity, E_p = 28,500 ksi [LRFD Art. 5.4.4.2]

Non Prestressed Reinforcement:
Yield Strength, f_y = 60,000 psi
Modulus of Elasticity, E_s = 29,000 ksi [LRFD Art. 5.4.3.2]

Unit weight of asphalt wearing surface = 140 pcf
T501 type barrier weight = 326 plf /side
Table A.2.4.1 Section Properties of AASHTO Type IV beam (notations as used in Figure A.2.4.1, Adapted from TxDOT Bridge Design Manual)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>W</th>
<th>y_t</th>
<th>y_b</th>
<th>Area</th>
<th>I</th>
<th>Wt/lf</th>
</tr>
</thead>
<tbody>
<tr>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
<td>in.²</td>
<td>in.³</td>
<td>lbs</td>
</tr>
<tr>
<td>20.00</td>
<td>26.00</td>
<td>8.00</td>
<td>54.00</td>
<td>9.00</td>
<td>23.00</td>
<td>6.00</td>
<td>8.00</td>
<td>8.00</td>
<td>29.25</td>
<td>24.75</td>
<td>788.40</td>
<td>260,403.00</td>
<td>821.00</td>
</tr>
</tbody>
</table>

where

- \( I \) = moment of inertia about the centroid of the non-composite Precast beam
- \( y_b \) = distance from centroid to the extreme bottom fiber of the non-composite precast beam
- \( y_t \) = distance from centroid to the extreme top fiber of the non-composite precast beam
- \( S_b \) = section modulus for the extreme bottom fiber of the non-composite precast beam = \( I/y_b = 260403.00/24.75 = 10521.33 \text{ in.}^3 \)
- \( S_t \) = section modulus for the extreme top fiber of the non-composite precast beam = \( I/y_t = 260403.00/29.25 = 8902.67 \text{ in.}^3 \)
A.2.4.2 Composite Section

A.2.4.2.1 Effective Flange Width

The effective flange width is lesser of:

\[
\text{1/4 span length: } \frac{108.583(12 \text{ in./ft})}{4} = 325.75 \text{ in.}
\]

Distance center to center of beams: \(8(12 \text{ in./ft}) = 96.00 \text{ in.} \) (controls)

12(Effective slab thickness) + greater of web thickness or \(\frac{1}{2}\) beam top flange width: \(12(8.0) + 1/2(20.0) = 106.00 \text{ in.}\)

Effective flange width = 96.00 in.

A.2.4.2.2 Modular Ratio between Slab and Beam Material

Following the TxDOT Design manual recommendation (Pg. #7-85) the modular ratio between slab and beam materials is taken as 1

\[
\frac{E_c \text{ for slab}}{E_c \text{ for beam}} = 1
\]

Transformed flange width = \(n \text{ (effective flange width)} = 1(96) = 96.0 \text{ in.}\)

Transformed Flange Area = \(n \text{ (effective flange width)} (t_s) = 1(96) (8) = 768.0 \text{ in.}^2\)

Table A.2.4.2 Properties of Composite Section

<table>
<thead>
<tr>
<th></th>
<th>Transformed Area in.²</th>
<th>(y_b) in.</th>
<th>(A_yb) in.</th>
<th>(A(y_{bc}-y_b)^2)</th>
<th>(I) in.⁴</th>
<th>(I+A(y_{bc}-y_b)^2) in.³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>788.40</td>
<td>24.75</td>
<td>19512.90</td>
<td>212231.53</td>
<td>260403.00</td>
<td>472634.53</td>
</tr>
<tr>
<td>Slab</td>
<td>768.00</td>
<td>58.00</td>
<td>44544.00</td>
<td>217868.93</td>
<td>4096.00</td>
<td>221964.93</td>
</tr>
<tr>
<td>Σ</td>
<td>1556.40</td>
<td>64056.90</td>
<td></td>
<td></td>
<td></td>
<td>694599.46</td>
</tr>
</tbody>
</table>

\(A_c = \text{total area of composite section} = 1556.4 \text{ in.}^2\)

\(h_c = \text{total height of composite section} = 62.0 \text{ in.}\)

\(I_c = \text{moment of inertia of composite section} = 694599.5 \text{ in.}^4\)

\(y_{bc} = \text{distance from the centroid of the composite section to extreme bottom fiber of the precast beam} = 64056.9/1556.4 = 41.157 \text{ in.}\)

\(y_{tg} = \text{distance from the centroid of the composite section to extreme top fiber of the precast beam} = 54 - 41.157 = 12.843 \text{ in.}\)

\(y_{tc} = \text{distance from the centroid of the composite section to extreme top fiber of the slab} = 62 - 41.157 = 20.843 \text{ in.}\)
S_{hc} = \text{composite section modulus for extreme bottom fiber of the precast beam} = \frac{I_c}{y_{hc}} = \frac{694599.50}{41.157} = 16876.83 \text{ in.}^3

S_{tg} = \text{composite section modulus for top fiber of the precast beam} = \frac{I_c}{y_{tg}} = \frac{694599.50}{12.843} = 54083.90 \text{ in.}^3

S_{tc} = \text{composite section modulus for top fiber of the slab} = \left(\frac{1}{n}\right)\frac{I_c}{y_{tc}} = \frac{1(694599.50)}{20.843} = 33325.31 \text{ in.}^3

The self weight of the beam and the weight of slab act on the non-composite simple span structure, while the weight of barriers, future wearing surface, and live load plus impact act on the composite simple span structure

[LRFD Art. 3.3.2]

DC = Dead load of structural components and non-structural attachments
Dead loads acting on the non-composite structure:
Self Weight of the beam = 0.821 kip/ft. (TxDOT Bridge Design Manual)

Weight of cast in place deck on each interior beam = (0.150 \text{pcf}) \left(\frac{8''}{12 \text{ in/ft}}\right) (8') = 0.800 \text{ kip/ft.}

Total Dead Load = 0.821 + 0.800 = 1.621 kips/ft.
AASHTO Type IV - LRFD Specifications

A.2.5.1.2 Super Imposed Dead Load

Dead loads placed on the composite structure:
The permanent loads on the bridge including loads from railing and wearing surface can be distributed uniformly among all beams if the following conditions are met: \[ \text{LRFD Art. 4.6.2.2.1} \]

- Width of deck is constant (O.K.)
- Number of beams, \( N_b \), is not less than four (\( N_b = 6 \)) (O.K.)
- Beams are parallel and have approximately the same stiffness (O.K.)
- The roadway part of the overhang, \( d_e \), is \( \leq 3.0 \) ft.
  \[ d_e = 3.0 - (\text{width of barrier at road surface}) = 3.0 - 1.417 = 1.583 \text{ ft.} \] (O.K.)
- Curvature in plan is less than \( 4^\circ \) (curvature = 0.0) (O.K.)
- Cross section of the bridge is consistent with one of the cross sections given in LRFD Table 4.6.2.2.1-1 (O.K.)

Since all the above criteria are satisfied, the barrier and wearing surface loads are equally distributed among the 6 beams.

Weight of T501 Rails or Barriers on each beam = \[
\frac{326 \text{ plf}}{1000} \div 6 \text{ beams} = 0.110 \text{ kips/ft./beam}
\]

Weight of 1.5” Wearing surface = \[
(0.140 \text{ kcf}) \left( \frac{1.5”}{12 \text{ in/ft}} \right) = 0.0175 \text{ kips/ft.}
\]

Weight of wearing surface on each beam = \[
\frac{(0.0175 \text{ ksf})(43.167 \text{ ft.})}{6 \text{ beams}} = 0.126 \text{ Kips/ft./beam}
\]

Total Super Imposed Dead Load = 0.110 + 0.126 = 0.236 kip/ft./beam

Shear forces and bending moments for the beam due to dead loads, superimposed dead loads at every tenth of the design span and at critical sections (hold down point or harp point) are shown in this section. The bending moment (M) and shear force (V) due to dead loads and superimposed dead loads at any section at a distance x are calculated using the following formulae.

\[
M = 0.5wx (L - x)
\]
\[
V = w(0.5L - x)
\]

As per the recommendations of TxDOT Bridge Design Manual Chap. 7, Sec 21

Distance of hold down point from centerline of bearing = \[
\frac{108.583}{2} - \frac{108.583}{20} = 48.862 \text{ ft.}
\]
### Table A.2.5.1. Shear forces and Bending moments due to Dead loads

<table>
<thead>
<tr>
<th>Distance x (ft.)</th>
<th>Section x/L</th>
<th>Beam Weight Shear Moment (kips)</th>
<th>Slab Weight Shear Moment (kips)</th>
<th>Barrier Shear (kips)</th>
<th>Wearing Surface Shear (kips)</th>
<th>Total Shear Moment (kips)</th>
<th>Shear Moment (K-ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.0</td>
<td>44.57</td>
<td>0.00</td>
<td>43.43</td>
<td>2.00</td>
<td>5.97</td>
<td>12.81</td>
</tr>
<tr>
<td>0.051</td>
<td>5.520</td>
<td>40.04</td>
<td>233.54</td>
<td>39.02</td>
<td>227.56</td>
<td>5.36</td>
<td>31.29</td>
</tr>
<tr>
<td>0.057</td>
<td>6.190</td>
<td>39.49</td>
<td>260.18</td>
<td>38.48</td>
<td>253.53</td>
<td>5.29</td>
<td>34.86</td>
</tr>
<tr>
<td>0.1</td>
<td>10.858</td>
<td>35.66</td>
<td>435.58</td>
<td>34.75</td>
<td>424.44</td>
<td>4.78</td>
<td>58.36</td>
</tr>
<tr>
<td>0.2</td>
<td>21.717</td>
<td>26.74</td>
<td>774.40</td>
<td>26.06</td>
<td>754.59</td>
<td>3.58</td>
<td>103.76</td>
</tr>
<tr>
<td>0.3</td>
<td>32.575</td>
<td>17.83</td>
<td>1016.38</td>
<td>17.37</td>
<td>990.38</td>
<td>2.39</td>
<td>136.18</td>
</tr>
<tr>
<td>0.4</td>
<td>43.433</td>
<td>8.91</td>
<td>1161.58</td>
<td>8.69</td>
<td>1131.86</td>
<td>1.19</td>
<td>155.63</td>
</tr>
<tr>
<td>0.45</td>
<td>48.862</td>
<td>4.46</td>
<td>1197.87</td>
<td>4.34</td>
<td>1167.24</td>
<td>0.60</td>
<td>160.49</td>
</tr>
<tr>
<td>0.5</td>
<td>54.292</td>
<td>0.00</td>
<td>1209.98</td>
<td>0.00</td>
<td>1179.03</td>
<td>0.00</td>
<td>162.11</td>
</tr>
</tbody>
</table>

**A.2.5.2 Shear Forces and Bending Moments due to Live Load**

**A.2.5.2.1 Live Load**

Design live load is HL-93 which consists of a combination of:

1. Design truck or design tandem with dynamic allowance [LRFD Art. 3.6.1.2.2]
   - The design truck is the same as HS20 design truck specified by the Standard Specifications, [STD Art. 3.6.1.2.2]. The design tandem consists of a pair of 25-kip axles spaced at 4.0 ft. apart. [LRFD Art. 3.6.1.2.3]
2. Design lane load of 0.64 kip/ft. without dynamic allowance [LRFD Art. 3.6.1.2.4]

**A.2.5.2.2 Live Load Distribution Factor for a Typical Interior Beam**

The bending moments and shear forces due to live load are determined using simplified distribution factor formulas, [LRFD Art. 4.6.2.2]. To use the simplified live load distribution factors the following conditions must be met [LRFD Art. 4.6.2.2.1]:

- Width of deck is constant (O.K.)
- Number of beams, N_b, is not less than four (N_b = 6) (O.K.)
- Beams are parallel and of the same stiffness (O.K.)
- The roadway part of the overhang, d_e ≤ 3.0 ft.
  - \(d_e = 3.0 - (\text{width of barrier at road surface}) = 3.0 - 1.417 = 1.583 \text{ ft.} \) (O.K.)
- Curvature in plan is less than 4\(^{\circ}\) (curvature = 0.0) (O.K.)

For precast concrete I-beams with cast-in-place concrete deck, the bridge type is (k) [LRFD Table 4.6.2.2.1-1]
The number of design lanes is computed as:
Number of design lanes = the integer part of the ratio of \((w/12)\), where \((w)\) is the clear roadway width, in ft., between the curbs [LRFD Art. 3.6.1.1.1]

\[ w = 43.167 \]

Number of design lanes = integer part of \((43.167/12)\) = 3 lanes

For all limit states except fatigue limit state:
For two or more lanes loaded:

\[
DFM = 0.075 + \left( \frac{S}{9.5} \right)^{0.6} \left( \frac{S}{L} \right)^{0.2} \left( \frac{K_g}{1200L^2t_s} \right)^{0.1}
\]

[LRFD Table 4.6.2.2.2b-1]

Provided that: 3.5 \( \leq \) \(S\) \( \leq \) 16; \(S = 8.0\) ft (O.K.)
4.5 \( \leq \) \(t_s\) \( \leq \) 12; \(t_s = 8.0\) in (O.K.)
20 \( \leq \) \(L\) \( \leq \) 240; \(L = 108.583\) ft. (O.K.)
\(N_b \geq 4; \quad N_b = 6\) (O.K.)
10,000 \( \leq \) \(K_g\) \( \leq \) 7,000,000 (O.K., as shown below)

where

\[ DFM = \text{distribution factor for moment for interior beam} \]
\[ S = \text{spacing of beams} = 8.0 \text{ ft} \]
\[ L = \text{beam span} = 108.583 \text{ ft} \]
\[ t_s = \text{depth of concrete deck} = 8.0 \text{ in.} \]
\[ K_g = \text{longitudinal stiffness parameter, in.}^4 \]
\[ K_g = n (1 + A e_g^2) \]

[LRFD Art. 3.6.1.1.1]

where

\[ n = \text{modular ratio between beam and slab materials} \]
\[ = \frac{E.(\text{beam})}{E.(\text{deck})} = 1 \]

The modular ratio is assumed to be 1 and needs to be updated once the optimum \(f_c'\) value is established, and the distribution factor based on new modular ratio will be compared to the present distribution factor and updated if needed

\[ A = \text{cross-sectional area of the beam (non-composite section)} \]
\[ A = 788.4 \text{ in.}^2 \]
\[ I = \text{moment of inertia of beam (non-composite section)} = 260,403.0 \text{ in.}^4 \]
\[ e_g = \text{distance between centers of gravity of the beam and slab, in.} \]
\[ = (t_s/2 + y_t) = (8/2 + 29.25) = 33.25 \text{ in.} \]
Therefore

\[ K_\ell = 1[260403 + 788.4(33.25)^2] = 1,132,028.48 \text{ in.}^4 \]

\[
DFM = 0.075 + \left( \frac{8}{9.5} \right)^{0.6} \left( \frac{8}{108.583} \right)^{0.2} \left( \frac{1132028.48}{12.0(108.583)(8)^{0.1}} \right) \\
= 0.075 + (0.902)(0.593)(1.054) = 0.639 \text{ lanes/beam}
\]

For one design lane loaded:

\[
DFM = 0.06 + \left( \frac{S}{14} \right)^{0.4} \left( \frac{S}{L} \right)^{0.3} \left( \frac{K_\ell}{12.0L_i} \right)^{0.1}
\]

\[
= 0.06 + \left( \frac{8}{14} \right)^{0.4} \left( \frac{8}{108.583} \right)^{0.3} \left( \frac{1132028.48}{12.0(108.583)(8)^{0.1}} \right)
\]

\[
= 0.06 + (0.8)(0.457)(1.054) = 0.445 \text{ lanes/beam}
\]

Thus, the case of two or more lanes loaded controls

\[ DFM = 0.639 \text{ lanes/beam} \]

For fatigue limit state:

LRFD Specifications, Art. 3.4.1, states that for fatigue limit state, a single design truck should be used. However, live load distribution factors given in LRFD Article 4.6.2.2 take into consideration the multiple presence factor, \( m \). LRFD Article 3.6.1.1.2 states that the multiple presence factor, \( m \), for one design lane loaded is 1.2. Therefore, the distribution factor for one design lane loaded with the multiple presence factor removed should be used. The distribution factor for the fatigue limit state is: 0.445/1.2 = 0.371 lanes/beam.

For two or more lanes loaded:

\[
DFV = 0.2 + \left( \frac{S}{12} \right) - \left( \frac{S}{35} \right)^2 \quad [\text{LRFD Table 4.6.2.2.3a-1}]
\]

Provided that: 

- \( 3.5 \leq S \leq 16 \); \( S = 8.0 \text{ ft} \) (O.K.)
- \( 4.5 \leq t_s \leq 12 \); \( t_s = 8.0 \text{ in} \) (O.K.)
- \( 20 \leq L \leq 240 \); \( L = 108.583 \text{ ft} \) (O.K.)

\[
N_b \geq 4; \quad N_b = 6 \quad \text{(O.K.)}
\]

where

\[ DFV = \text{Distribution factor for shear for interior beam} \]

\[ S = \text{Beam spacing} = 8 \text{ ft.} \]

Therefore the distribution factor for shear force is:

\[
DFV = 0.2 + \left( \frac{8}{12} \right) - \left( \frac{8}{35} \right)^2 = 0.814 \text{ lanes/beam}
\]
For one design lane loaded:

$$DFV = 0.36 + \left( \frac{S}{25.0} \right) = 0.36 + \left( \frac{8}{25.0} \right) = 0.68 \text{ lanes/beam} \quad \text{[LRFD Table 4.6.2.2.3a-1]}$$

Thus, the case of two or more lanes loaded controls

$$DFV = 0.814 \text{ lanes/beam}$$

**A.2.5.2.3 Dynamic Allowance**

$$IM = 33\% \quad \text{[LRFD Table 3.6.2.1-1]}$$

where

$$IM = \text{dynamic load allowance, applied to truck load only}$$

For all limit states except for fatigue limit state:

Shear force and bending moment due to truck load on a per-lane-basis are calculated at tenth-points of the span using the following equations

For $x/L = 0 - 0.333$

Maximum bending moment due to truck load, $M = \frac{72(x)[(L - x) - 9.33]}{L}$

For $x/L = 0.333 - 0.5$

Maximum bending moment due to truck load, $M = \frac{72(x)[(L - x) - 4.67]}{L} - 112$

Unfactored Bending Moment due to truck load plus impact

$$M_{LT} = (\text{bending moment per lane})(DFM)(1+IM)$$

$$= (\text{bending moment per lane})(0.639)(1+0.33)$$

$$= (\text{bending moment per lane})(0.85)$$

For $x/L = 0 - 0.5$

Maximum Shear Force due to truck load, $V = \frac{72[(L - x) - 9.33]}{L}$

Unfactored Shear Force due to truck load plus impact

$$V_{LT} = (\text{shear force per lane})(DFV)(1+IM)$$

$$= (\text{shear force per lane})(0.814)(1+0.33)$$

$$= (\text{shear force per lane})(1.083)$$

$M_{LT}$ and $V_{LT}$ values at every tenth of span are shown in Table A.2.5.2
For fatigue limit state:
Art. 3.6.1.4.1 in the LRFD specifications states that the fatigue load is a single design truck which has the same axle weight used in all other limit states but with a constant spacing of 30.0 ft. between the 32.0 kip axles. Bending moment envelope on a per-lane-basis is calculated using the following equation:

For $x/L = 0 - 0.241$

Maximum bending moment due to fatigue truck load, $M = \frac{72(x)((L - x) - 18.22)}{L}$

For $x/L = 0.241 - 0.5$

$M = \frac{72(x)((L - x) - 11.78)}{L} - 112$

Unfactored Bending Moment due to fatigue truck load plus impact

$M_f = (\text{bending moment per lane}) (DFM) (1+IM)$

$= (\text{bending moment per lane})(0.371)(1+0.15)$

$= (\text{bending moment per lane})(0.427)$

$M_f$ values at every tenth of span are shown in Table A.2.5.2

The maximum bending moments and shear forces due to uniformly distributed lane load of 0.64 kip/ft. are calculated using the following formulae:

Maximum Bending moment, $M_x = 0.5(0.64)(x)(L-x)$

Unfactored Bending Moment due to lane load

$M_{LL} = (\text{bending moment per lane}) (DFM)$

$= (\text{bending moment per lane})(0.639)$

Maximum Shear Force, $V_x = \frac{0.32(L-x)^2}{L}$ for $x \leq 0.5L$

Unfactored Shear Force due to lane load

$V_{LL} = (\text{shear force per lane})(DFV)$

$= (\text{shear force per lane})(0.814)$

where

$x = \text{distance from the support to the section at which bending moment or shear force is calculated}$

$L = \text{span length} = 108.583 \text{ ft.}$
Table A.2.5.2. Shear forces and Bending moments due to Live loads

<table>
<thead>
<tr>
<th>Distance x</th>
<th>Section x/L</th>
<th>Truck Loading</th>
<th>Lane loading</th>
<th>Fatigue Truck loading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Truck Load</td>
<td>Truck load + Impact</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shear</td>
<td>Moment</td>
<td>Shear</td>
</tr>
<tr>
<td></td>
<td></td>
<td>V_LT</td>
<td>M_LT</td>
<td>V_LL</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>65.81</td>
<td>0.00</td>
<td>71.25</td>
</tr>
<tr>
<td>5.520</td>
<td>0.051</td>
<td>62.15</td>
<td>343.09</td>
<td>67.28</td>
</tr>
<tr>
<td>6.190</td>
<td>0.057</td>
<td>61.71</td>
<td>381.98</td>
<td>66.81</td>
</tr>
<tr>
<td>10.858</td>
<td>0.1</td>
<td>58.61</td>
<td>636.43</td>
<td>63.45</td>
</tr>
<tr>
<td>21.717</td>
<td>0.2</td>
<td>51.41</td>
<td>1116.54</td>
<td>55.66</td>
</tr>
<tr>
<td>32.575</td>
<td>0.3</td>
<td>44.21</td>
<td>1440.25</td>
<td>47.86</td>
</tr>
<tr>
<td>43.433</td>
<td>0.4</td>
<td>37.01</td>
<td>1629.82</td>
<td>40.07</td>
</tr>
<tr>
<td>48.862</td>
<td>0.45</td>
<td>33.41</td>
<td>1671.64</td>
<td>36.17</td>
</tr>
<tr>
<td>54.292</td>
<td>0.5</td>
<td>29.81</td>
<td>1674.37</td>
<td>32.27</td>
</tr>
</tbody>
</table>

A.2.5.3 Load Combinations

Total factored load shall be taken as:

\[ Q = \eta \sum \gamma_i q_i \]  

[LRFD Eq. 3.4.1-1]

where

\[ \eta = \text{a factor relating to ductility, redundancy and operational importance} \]  

[LRFD Art. 1.3.2]

\[ \gamma_i = \text{load factors} \]  

[LRFD Table 3.4.1-1]

\[ q_i = \text{specified loads} \]

Investigating different limit states given in LRFD Art. 3.4.1, the following limit states are applicable in present case:

Service I: check compressive stresses in prestressed concrete components:

\[ Q = 1.00(DC + DW) + 1.00(LL+IM) \]  

[LRFD Table 3.4.1-1]

This load combination is the general combination for service limit state stress checks and applies to all conditions other than Service III.

Service III: check tensile stresses in prestressed concrete components:

\[ Q = 1.00(DC + DW) + 0.80(LL+IM) \]  

[LRFD Table 3.4.1-1]

This load combination is a special combination for service limit state stress checks that applies only to tension in prestressed concrete structures to control cracks.
AASHTO Type IV - LRFD Specifications

Strength I: check ultimate strength:  
[LRFD Table 3.4.1-1 and 2]
Maximum Q = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)
Minimum Q = 0.90(DC) + 0.65(DW) + 1.75(LL + IM)
This load combination is the general load combination for strength limit state design.

For simple span bridges, the maximum load factors produce maximum effects. However, minimum load factors are used for dead load (DC), and wearing surface load (DW) when dead load and wearing surface stresses are opposite to those of live load.

Fatigue: check stress range in strands:  
[LRFD Table 3.4.1-1]
Q = 0.75(LL + IM)
This load combination is a special load combination to check the tensile stress range in the strands due to live load and dynamic load allowance

where
DC = self weight of beam
DW = wearing surface load
LL = live load
IM = Dynamic allowance

A.2.6
ESTIMATION OF REQUIRED PRESTRESS

A.2.6.1
Service load Stresses at Midspan

The required number of strands is usually governed by concrete tensile stresses at the bottom fiber for load combination at Service III at the section of maximum moment or at the harp points. For estimating the number of strands, only the stresses at midspans are considered.

Bottom tensile stresses at midspan due to applied dead and live loads using load combination Service III is:

\[
f_b = \frac{M_g + M_s}{S_b} + \frac{M_{SDL} + (0.8)(M_{LT} + M_{LL})}{S_{bc}}
\]

where
\(f_b\) = concrete stress at the bottom fiber of the beam
\(M_g\) = Unfactored bending moment due to beam self-weight
\(M_s\) = Unfactored bending moment due to slab weight
\(M_{SDL}\) = Unfactored bending moment due to super imposed dead load
\(M_{LT}\) = Bending moment due to truck load plus impact
\(M_{LL}\) = Bending moment due to lane load
Substituting the bending moments and section modulus values, bottom fiber stresses at mid span is:

\[ f_{bc} = \frac{(1209.98 + 1179.03)(12)}{10521.33} + \frac{(347.81)(12) + (0.8)(1423.00 + 602.72)(12)}{16876.83} \]

\[ = 2.725 + 1.400 = 4.125 \text{ ksi} \]

### A.2.6.2 Allowable Stress Limit

At service load conditions, allowable tensile stress is

\[ F_b = 0.19 \sqrt{f_{c}'} \quad \text{[LRFD Art. 5.9.4.2b]} \]

where \( f_{c}' \) = beam concrete strength at service, ksi

\[ F_b = 0.19 \sqrt{5} = -0.425 \text{ ksi} \]

### A.2.6.3 Required Number of Strands

Required precompressive stress in the bottom fiber after losses:

Bottom tensile stress – allowable tensile stress at final = \( f_b - F_b \)

\[ f_{pb} = 4.125 - 0.425 = 3.700 \text{ ksi} \]

Assuming the distance from the center of gravity of strands to the bottom fiber of the beam is equal to \( y_{bs} = 2 \) in.

Strand Eccentricity at midspan:

\[ e_c = y_b - y_{bs} = 24.75 - 2 = 22.75 \text{ in.} \]

Bottom fiber stress due to prestress after losses:

\[ f_b = \frac{P_{pe} + P_{pe} e_c}{A_{So}} \]

where \( P_{pe} = \) effective prestressing force after all losses

\[ 3.700 = \frac{22.75 P_{pe}}{788.4} + \frac{10521.33 P_{pe}}{10521.33} \]

Solving for \( P_{pe} \) we get,

\[ P_{pe} = 1078.5 \text{ Kips} \]

Assuming final losses = 20% of \( f_{pi} \)

Assumed final losses = 0.2(202.5) = 40.5 ksi

The prestress force per strand after losses

\[ = (\text{cross sectional area of one strand}) \left[ f_{pi} - \text{losses} \right] \]

\[ = 0.153(202.5 - 40.5) = 24.78 \text{ Kips} \]

Number of Strands Required = \( 1078.5/24.78 = 43.52 \)

Try 44 – \( \frac{1}{2} \) in. diameter, 270 ksi strands as an initial trial

---

Detailed Design Examples - 14
Strand eccentricity at midspan after strand arrangement

\[ e_c = 24.75 - \frac{12(2+4+6) + 8(8)}{44} = 20.02 \text{ in.} \]

\[ P_{pe} = 44(24.78) = 1090.32 \text{ Kips} \]

\[ f_b = \frac{1090.32}{788.4} + \frac{20.02(1090.32)}{10521.33} \]
\[ = 1.383 + 2.075 = 3.458 \text{ ksi} < f_{pb \text{ reqd.}} = 3.700 \text{ ksi} \quad \text{(N.G.)} \]

Therefore try 46 strands

Effective strand eccentricity at midspan after strand arrangement

\[ e_c = 24.75 - \frac{12(2+4+6) + 10(8)}{46} = 19.88 \text{ in.} \]

\[ P_{pe} = 46(24.78) = 1139.88 \text{ Kips} \]

\[ f_b = \frac{1139.88}{788.4} + \frac{19.88(1139.88)}{10521.33} \]
\[ = 1.446 + 2.154 = 3.600 \text{ ksi} < f_{pb \text{ reqd.}} = 3.700 \text{ ksi} \quad \text{(N.G.)} \]

Therefore try 48 strands

Effective strand eccentricity at midspan after strand arrangement

\[ e_c = 24.75 - \frac{12(2+4+6) + 10(8)+2(10)}{48} = 19.67 \text{ in.} \]

\[ P_{pe} = 48(24.78) = 1189.44 \text{ Kips} \]

\[ f_b = \frac{1189.44}{788.4} + \frac{19.67(1189.44)}{10521.33} \]
\[ = 1.509 + 2.223 = 3.732 \text{ ksi} > f_{pb \text{ reqd.}} = 3.700 \text{ ksi} \quad \text{(O.K.)} \]

Therefore use 48 strands
A.2.7 PRESTRESS LOSSES

Total prestress loss:

\[ \Delta f_{PT} = \Delta f_{ESP} + \Delta f_{SR} + \Delta f_{CR} + \Delta f_{R2} \]  

[LRFD Eq. 5.9.5.1-1]

where

- \( \Delta f_{ESP} \): loss of prestress due to elastic shortening
- \( \Delta f_{SR} \): loss of prestress due to concrete shrinkage
- \( \Delta f_{CR} \): loss of prestress due to creep of concrete
- \( \Delta f_{R2} \): loss of prestress due to relaxation of steel after transfer

Number of strands = 48

A number of iterations will be performed to arrive at the optimum \( f'_c \) and \( f'_{ci} \)

A.2.7.1 Iteration 1

A.2.7.1.1 Elastic Shortening

\[ \Delta f_{ESP} = \frac{E_p}{E_{ci}} f_{cgp} \]  

[LRFD Art. 5.9.5.2.3a]

where

- \( f_{cgp} \): sum of concrete stresses at the center of gravity of the prestressing steel due to prestressing force at transfer and self weight of the member at sections of maximum moment

LRFD Art. 5.9.5.2.3a states that \( f_{cgp} \) can be calculated on the basis of prestressing steel stress assumed to be 0.7\( f_{pu} \) for low-relaxation strands. However, common practice assumes the initial losses as a percentage of the initial prestressing stress before release, \( f_{pi} \). In both procedures initial losses assumed should be checked.
and if different from the assumed value a second iteration should be carried out. In this example, 8% $f_{pi}$ initial loss is used

\[ f_{gp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_e) e_c}{I} \]

where

- $P_i = \text{pretension force after allowing for the initial losses, assuming 8\% initial losses} = (\text{number of strands})(\text{area of each strand})(0.92(f_{pi}))$
- $M_e = \text{Unfactored bending moment due to beam self weight} = 1209.98 \text{ K-ft.}$
- $e_c = \text{eccentricity of the strand at the midspan} = 19.67 \text{ in.}$

\[ f_{gp} = \frac{1368.19}{788.4} + \frac{1368.19(19.67)}{260403} - \frac{1209.98(12)(19.67)}{260403} = 1.735 + 2.033 - 1.097 = 2.671 \text{ ksi} \]

$E_{ci} = \text{modulus of elasticity of beam at release} = (w_c)^{1.5}(33) \sqrt{f_{ci}'}$

where

- $w_c = \text{weight of concrete} = 150 \text{ pcf}$
- Initial assumed value of $f_{ci}' = 4000 \text{ psi}$

\[ E_{ci} = (150)^{1.5}(33) \sqrt{4000}^{1000} = 3834 \text{ ksi} \]

$E_p = \text{modulus of elasticity of prestressing reinforcement} = 28500 \text{ ksi}$

Therefore loss due to elastic shortening:

\[ \Delta f_{pES} = \frac{28500}{3834} (2.671) = 19.855 \text{ ksi} \]

A.2.7.1.2 Shrinkage

\[ \Delta f_{pSR} = 17 - 0.15 H \]

where $H$ is relative humidity = 60%

\[ \Delta f_{pSR} = [17 - 0.15(60)] = 8.0 \text{ ksi} \]

A.2.7.1.3 Creep of Concrete

\[ \Delta f_{pCR} = 12f_{gp} - 7\Delta f_{cdp} \]

where $\Delta f_{cdp} = \text{change of stresses at the center of gravity of the prestressing steel due to permanent loads except the dead load present at the time the prestress force is applied calculated at the same section as } f_{gp}$
A.2.7.1.4 Relaxation of Prestressing Strands

A.2.7.1.4.1 Relaxation before Transfer

Initial loss due to relaxation of prestressing steel is accounted for in the beam fabrication process. Therefore, loss due to relaxation of the prestressing steel prior to transfer is not computed, i.e. \( \Delta f_{pR1} = 0 \). Recognizing this for pretensioned members, LRFD Article 5.9.5.1 allows the portion of the relaxation loss that occurs prior to transfer to be neglected in computing the final loss.

A.2.7.1.4.2 Relaxation after Transfer

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer is

\[
\Delta f_{pR2} = 30\% \left[ 20.0 - 0.4 \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR}) \right] = 0.3 \left[ 20.0 - 0.4(19.855) - 0.2(8 + 23.05) \right] = 1.754 \text{ ksi}
\]

TxDOT Bridge Design Manual (Pg. # 7-85) recommends that 50\% of the final steel relaxation loss be considered for calculation of total initial loss given as

\[
[\text{Elastic shortening loss} + 0.5(\text{total steel relaxation loss})]
\]

Initial Prestress loss % = \( \frac{\Delta f_{pES} + 0.5\Delta f_{pR2}}{f_{pi}} \)\times 100

\[
= \frac{[19.855 + 0.5(1.754)]}{202.5} = 10.24\% > 8\% \text{ (assumed initial prestress losses)}
\]

Therefore next trial is required assuming 10.24\% initial losses.

\[
\Delta f_{cdp} = \frac{M_{Sc}}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}
\]

where

- \( M_S \) = slab moment = 1179.03 K-ft.
- \( M_{SDL} \) = superimposed dead load moment = 347.81 K-ft.
- \( y_{bc} \) = 41.157 in.
- \( y_{bs} \) = the distance from center of gravity of the strand at midspan to the bottom of the beam = 24.75 – 19.67 = 5.08 in.
- \( I \) = moment of inertia of the non-composite section = 260403 in.\(^4\)
- \( I_c \) = moment of inertia of composite section = 694599.5 in.\(^4\)

\[
\Delta f_{cdp} = \frac{1179.03(12)(19.67)}{260403} + \frac{(347.81)(12)(41.157 - 5.08)}{694599.5} = 1.069 + 0.217 = 1.286 \text{ ksi}
\]

\[
\Delta f_{pCR} = 12(2.671) - 7(1.286) = 23.05 \text{ ksi}
\]
\[ \Delta f_{p_{ES}} = \frac{E_p}{E_{ci}} f_{egp} \]  
[LRFD Art. 5.9.5.2.3a]

where

\[ f_{egp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g) e_c}{1} \]

\( P_i \) = pretension force after allowing for the initial losses, assuming 10.24% initial losses
= (number of strands)(area of each strand)(0.8976(f_{pi}))
= 48(0.153)(0.8976)(202.5) = 1334.87 Kips

\( M_g = 1209.98 \) K-ft.
\( e_c = 19.67 \) in.

\[ f_{egp} = \frac{1334.87}{788.4} + \frac{1334.87(19.67)^2}{260403} - \frac{1209.98(12)(19.67)}{260403} = 1.693 + 1.983 - 1.097 = 2.579 \text{ ksi} \]

\( E_{ci} = 3834 \) ksi
\( E_p = 28500 \) ksi

Therefore loss due to elastic shortening:

\[ \Delta f_{p_{ES}} = \frac{28500}{3834} (2.579) = 19.17 \text{ ksi} \]

\[ \Delta f_{p_{CR}} = 12f_{egp} - 7\Delta f_{ cd} \]  
[LRFD Eq. 5.9.5.4.3-1]

where

\( f_{egp} = 2.579 \) ksi
\( \Delta f_{ cd} = 1.286 \) ksi
\( \Delta f_{p_{CR}} = 12(2.579) - 7(1.286) = 21.95 \) ksi.
\( \Delta f_{pSR} = 8.0 \) ksi

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

\[ \Delta f_{pR2} = 30\% [20.0 - 0.4 \Delta f_{p_{ES}} - 0.2(\Delta f_{pSR} + \Delta f_{p_{CR}}) \]
= 0.3[20.0 - 0.4(19.17) - 0.2(8 + 21.95)] = 1.903 ksi

Initial Prestress loss % = \( \frac{(\Delta f_{p_{ES}} + 0.5\Delta f_{pR2})100}{f_{pi}} \)
= \( \frac{[19.17 + 0.5(1.903)]100}{202.5} = 9.94\% < 10.24\% \) (assumed initial prestress losses)
Therefore next trial is required assuming 9.94% initial losses

\[
\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cp} \\
\text{[LRFD Art. 5.9.5.2.3a]}
\]

where

\[
f_{cp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g) e_c}{I}
\]

\[P_i = \text{pretension force after allowing for the initial losses, assuming 9.94\% initial losses} = (\text{number of strands}) (\text{area of each strand}) [0.9006(f_{pi})]
\]

\[= 48(0.153)(0.9006)(202.5) = 1339.34 \text{ Kips}
\]

\[M_g = 1209.98 \text{ K-ft.}
\]

\[e_c = 19.67 \text{ in.}
\]

\[
f_{cp} = \frac{1339.34}{788.4} + \frac{1339.34(19.67)^2}{260403} - \frac{1209.98(12)(19.67)}{260403}
\]

\[= 1.699 + 1.99 - 1.097 = 2.592 \text{ ksi}
\]

\[E_{ci} = 3834 \text{ ksi}
\]

\[E_p = 28500 \text{ ksi}
\]

Therefore loss due to elastic shortening:

\[
\Delta f_{pES} = \frac{28500}{3834} (2.592) = 19.27 \text{ ksi}
\]

\[
\Delta f_{pCR} = 12f_{cp} - 7\Delta f_{cp} \\
\text{[LRFD Eq. 5.9.5.4.3-1]}
\]

where

\[f_{cp} = 2.592 \text{ ksi}
\]

\[\Delta f_{cp} = 1.286 \text{ ksi}
\]

\[\Delta f_{pCR} = 12(2.592) - 7(1.286) = 22.1 \text{ ksi.}
\]

\[\Delta f_{pSR} = 8.0 \text{ ksi}
\]

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

\[
\Delta f_{pR2} = 30\% [20.0 - 0.4 \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})]
\]

\[= 0.3[20.0 - 0.4(19.27) - 0.2(8 + 22.1)] = 1.882 \text{ ksi}
\]

Initial Prestress loss % = \[
\frac{(\Delta f_{pES} + 0.5\Delta f_{pR2})100}{f_{pi}}
\]

\[= \frac{[19.27 + 0.5(1.882)]100}{202.5} = 9.98\% \approx 9.94\% \text{ (assumed initial prestress losses)}
\]
A.2.7.1.5  Total Losses at Transfer

Total Initial loss, $\Delta f_{pi} = (\Delta f_{pES} + 0.5 \Delta f_{pR2}) = [19.27 + 0.5(1.882)] = 20.21$ ksi

$f_{pi} =$ effective initial prestress $= 202.5 - 20.21 = 182.29$ ksi

$P_1 =$ effective pretension force after allowing for the initial losses

$= 48(0.153)(182.29) = 1338.74$ Kips

A.2.7.1.6  Total Losses at Service Loads

$\Delta f_{pES} = 19.270$ ksi

$\Delta f_{pSR} = 8.000$ ksi

$\Delta f_{pCR} = 22.100$ ksi

$\Delta f_{pR2} = 1.882$ ksi

Total final loss, $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2} \quad [LRFD \ Eq. \ 5.9.5.1-1]$

$= 19.270 + 8.000 + 22.100 + 1.882 = 51.25$ ksi

$= \frac{51.25(100)}{202.5} = 25.31 \%$

$f_{pe} =$ effective final prestress $= f_{pi} - \Delta f_{pT} = 202.5 - 51.25 = 151.25$ ksi

Check Prestressing stress limit at service limit state:

$f_{pe} \leq 0.8f_{py} = 0.8(243) = 194.4$ ksi $> 151.25$ ksi  (O.K.)

Total prestressing force after all losses

$P_{pe} = 48(0.153)(151.25) = 1110.78$ Kips

A.2.7.1.7  Final Stresses at midspan

Final stress in the bottom fiber at midspan:

$f_b = \frac{P_{pe} + P_{pc} e_c}{A_{s_b}}$

$f_b = \frac{1110.78 + 19.47(1110.78)}{788.4 + 10521.33} $

$= 3.486$ ksi $< f_{pb \ reqd.} = 3.700$ ksi  (N.G)

Therefore try 50 strands

$e_c = 24.75 - \frac{12(2+4+6) + 10(8) + 4(10)}{50} = 19.47$ in.

$P_{pe} = 50(0.153)(151.25) = 1157.06$ Kips

$f_b = \frac{1157.06 + 19.47(1157.06)}{788.4 + 10521.33} $

$= 3.609$ ksi $< f_{pb \ reqd.} = 3.700$ ksi  (N.G.)
Therefore try 52 strands

\[ e_c = 24.75 - \frac{12(2+4+6) + 10(8) + 6(10)}{52} = 19.29 \text{ in.} \]

\[ P_{pe} = 52(0.153)(151.25) = 1203.35 \text{ Kips} \]

\[ f_b = \frac{1203.35}{788.4} + \frac{19.29(1203.35)}{10521.33} = 1.526 + 2.206 = 3.732 \text{ ksi} > f_{pb \text{ reqd.}} = 3.700 \text{ ksi} \quad \text{(O.K.)} \]

Therefore use 52 strands

Concrete stress at the top fiber of the beam, \( f_t \)

Under permanent loads, Service I:

\[ f_t = \frac{P_{pe}}{A} - \frac{P_{pe} e_c}{S_t} + \frac{M_g}{S_t} + \frac{M_{SDL}}{S_{tg}} \]

\[ = \frac{1203.35}{788.4} - \frac{19.29(1203.35)}{8902.67} + \frac{(1209.98 + 1179.03)(12)}{8902.67} + \frac{347.81(12)}{54083.9} \]

\[ = 1.526 - 2.607 + 3.22 + 0.077 = 2.216 \text{ ksi} \]

Under permanent and transient loads, Service I:

\[ f_t = \frac{P_{pe}}{A} - \frac{P_{pe} e_c}{S_t} + \frac{M_g}{S_t} + \frac{M_{SDL}}{S_{tg}} + \frac{(M_{LT} + M_{LL})}{S_{tg}} \]

\[ = 2.216 + \frac{(1423 + 602.72)12}{54083.9} = 2.216 + 0.449 = 2.665 \text{ ksi} \]

A.2.7.1.8

Initial Prestress

\[ P_t = 52(0.153)(182.29) = 1450.30 \text{ Kips} \]

Initial concrete stress at top fiber of the beam at hold down point

\[ f_{it} = \frac{P_t}{A} - \frac{P_t e_c}{S_t} + \frac{M_g}{S_t} \]

where

\[ M_g = \text{Moment due to beam self weight at hold down point} = 1197.87 \text{ K-ft.} \]

\[ f_{it} = \frac{1450.3}{788.4} - \frac{19.29(1450.3)}{8902.67} + \frac{1197.87(12)}{8902.67} \]

\[ = 1.839 - 3.142 + 1.615 = 0.312 \text{ ksi} \]
A.2.7.2

Iteration 2

A.2.7.2.1

Elastic Shortening

Initial concrete stress at bottom fiber of the beam at hold down point

\[
f_{bi} = \frac{P_i}{A} + \frac{P_i \epsilon_c}{S_b} - \frac{M_e}{S_b}
\]

\[
f_{bi} = \frac{1450.3}{788.4} + \frac{19.29(1450.3)}{10521.33} - \frac{1197.87(12)}{10521.33}
\]

\[
= 1.839 + 2.659 - 1.366 = 3.132 \text{ ksi}
\]

Compression stress limit at transfer is 0.6\(f'_{ci}\)

Therefore, \(f'_{ci} \text{ reqd.} = \frac{3132}{0.6} = 5220 \text{ psi}\)

Number of strands = 52

\[
\Delta f_{pres} = \frac{E_{p}}{E_{ci}} f_{cgp}
\]

where

\(f_{cgp} = \text{sum of concrete stresses at the center of gravity of the prestressing steel due to prestressing force at transfer and self weight of the member at sections of maximum moment}\)

\[
f_{cgp} = \frac{P_i}{A} + \frac{P_i \epsilon_c^2}{I} - \frac{(M_e) \epsilon_c}{I}
\]

where

\(P_i = \text{pretension force after allowing for the initial losses, assuming 9.98\% initial losses} = \text{(number of strands)(area of each strand)(0.9002)(f_{pi})}\)

\(= 52(0.153)(0.9002)(202.5) = 1450.3 \text{ Kips}\)

\(M_e = \text{Unfactored bending moment due to beam self weight} = 1209.98 \text{ K-ft.}\)

\(\epsilon_c = \text{eccentricity of the strand at the midspan} = 19.29 \text{ in.}\)

\[
f_{cgp} = \frac{1450.3}{788.4} + \frac{1450.3(19.29)^2}{260403} - \frac{1209.98(12)(19.29)}{260403}
\]

\[
= 1.839 + 2.072 - 1.076 = 2.835 \text{ ksi}
\]

\(E_{ci} = \text{modulus of elasticity of beam at release} = (w_c)^{1.5}(33)\sqrt{f'_{ci}}\)

where

\(w_c = \text{weight of concrete} = 150 \text{ pcf}\)

\(f'_{ci} = 5220 \text{ psi}\)

\(E_{ci} = (150)^{1.5}(33)\sqrt{5220} \frac{1}{1000} = 4380.12 \text{ ksi}\)
A.2.7.2.2 Shrinkage

\[ \Delta f_{pSR} = 17 - 0.15 H \quad \text{[LRFD Eq. 5.9.5.4.2-1]} \]

where \( H \) is relative humidity = 60%

\[ \Delta f_{pSR} = [17 - 0.15(60)] = 8.0 \text{ ksi} \]

A.2.7.2.3 Creep of Concrete

\[ \Delta f_{pCR} = 12f_{cgp} - 7f_{cdp} \quad \text{[LRFD Eq. 5.9.5.4.3-1]} \]

where

\[ f_{cdp} = \text{change of stresses at the center of gravity of the prestressing steel due to permanent loads except the dead load present at the time the prestress force is applied calculated at the same section as } f_{cgp} \]

\[ f_{cdp} = \frac{M_{S}e_{c}}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_{c}} \]

where

\[ M_{S} = \text{slab moment} = 1179.03 \text{ K-ft.} \]

\[ M_{SDL} = \text{superimposed dead load moment} = 347.81 \text{ K-ft.} \]

\[ y_{bc} = 41.157 \text{ in.} \]

\[ y_{bs} = \text{the distance from center of gravity of the strand at midspan to the bottom of the beam} = 24.75 - 19.29 = 5.46 \text{ in.} \]

\[ I_{n} = \text{moment of inertia of the non-composite section} = 260403 \text{ in.}^{4} \]

\[ I_{c} = \text{moment of inertia of composite section} = 694599.5 \text{ in.}^{4} \]

\[ f_{cdp} = \frac{1179.03(12)(19.29)}{260403} + \frac{(347.81)(12)(41.157 - 5.46)}{694599.5} \]

\[ f_{cdp} = 1.048 + 0.214 = 1.262 \text{ ksi} \]

\[ \Delta f_{pCR} = 12(2.835) - 7(1.262) = 25.19 \text{ ksi.} \]

A.2.7.2.4 Relaxation of Prestressing Strands

A.2.7.2.4.1 Relaxation before Transfer

Loss due to Relaxation of steel before transfer

\[ \Delta f_{pR1} = 0 \]
A.2.7.2.4.2 Relaxation after Transfer

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

\[ \Delta f_{PR2} = 30\% \left[ 20.0 - 0.4 \Delta f_{PES} - 0.2(\Delta f_{PSR} + \Delta f_{PCR}) \right] \]

\[ = 0.3 \left[ 20.0 - 0.4(18.45) - 0.2(8 + 25.19) \right] = 1.795 \text{ ksi} \]

TDOT Bridge Design Manual (Pg. # 7-85) recommends that 50% of the final steel relaxation loss be considered for calculation of total initial loss given as [Elastic shortening loss + 0.50(total steel relaxation loss)]

Initial Prestress loss % = \( \frac{(\Delta f_{PES} + 0.5\Delta f_{PR2})}{f_{pi}} \times 100 \)

\[ = \frac{[18.45 + 0.5(1.795)]100}{202.5} = 9.55\% < 9.98\% \text{ (assumed initial prestress losses)} \]

Therefore next trial is required assuming 9.55% initial losses

\[ \Delta f_{PES} = \frac{E_p}{E_{ci}} \frac{f_{cp}}{f_{p}} \]

[LRFD Art. 5.9.5.2.3a]

where

\[ f_{cgp} = \frac{P_i + P_i e^2}{A} - \frac{(M_g) e}{I} \]

\[ P_i = \text{pretension force after allowing for the initial losses, assuming 9.55\% initial losses} = (\text{number of strands})(\text{area of each strand})(0.9045(f_{pi})) \]

\[ = 52(0.153)(0.9045)(202.5) = 1457.23 \text{ Kips} \]

\[ M_g = 1209.98 \text{ K-ft.} \]

\[ e_c = 19.29 \text{ in.} \]

\[ f_{cgp} = \frac{1457.23 + 1457.23(19.29)^2}{788.4} - \frac{1209.98(12)(19.29)}{260403} \]

\[ = 1.848 + 2.082 - 1.076 = 2.854 \text{ ksi} \]

\[ E_c = 4380.12 \text{ ksi} \]

\[ E_p = 28500 \text{ ksi} \]

Therefore loss due to elastic shortening:

\[ \Delta f_{PES} = \frac{28500}{4380.12} (2.854) = 18.57 \text{ ksi} \]
AASHTO Type IV - LRFD Specifications

\[ \Delta f_{pCR} = 12f_{g} - 7\Delta f_{dp} \]  
\[ \text{[LRFD Eq. 5.9.5.4.3-1]} \]

where

\[ f_{g} = 2.854 \text{ ksi} \]
\[ \Delta f_{dp} = 1.262 \text{ ksi} \]
\[ \Delta f_{pCR} = 12(2.854) - 7(1.262) = 25.41 \text{ ksi.} \]
\[ \Delta f_{pSR} = 8.0 \text{ ksi} \]

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

\[ \Delta f_{pR2} = 30\% \left[ 20.0 - 0.4 \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR}) \right] \]
\[ = 0.3 \left[ 20.0 - 0.4(18.57) - 0.2(8 + 25.41) \right] = 1.767 \text{ ksi} \]

Initial Prestress loss % = \[ \frac{(\Delta f_{pES} + 0.5\Delta f_{pR2})100}{f_{pi}} \]
\[ = \frac{[18.57 + 0.5(1.767)]100}{202.5} = 9.61\% \approx 9.55\% \text{ (assumed initial prestress losses)} \]

Total Initial loss, \[ \Delta f_{pi} = (\Delta f_{pES} + 0.5\Delta f_{pR2}) - [18.57 + 0.5(1.767)] = 19.45 \text{ ksi} \]
\[ f_{pi} = \text{effective initial prestress} = 202.5 - 19.45 = 183.05 \text{ ksi} \]
\[ P_{i} = \text{effective pretension force after allowing for the initial losses} \]
\[ = 52(0.153)(183.05) = 1456.35 \text{ Kips} \]

\[ \Delta f_{pES} = 18.570 \text{ ksi} \]
\[ \Delta f_{pSR} = 8.000 \text{ ksi} \]
\[ \Delta f_{pCR} = 25.410 \text{ ksi} \]
\[ \Delta f_{pR2} = 1.767 \text{ ksi} \]

Total final loss, \[ \Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2} \]  
\[ \text{[LRFD Eq. 5.9.5.1-1]} \]
\[ = 18.570 + 8.000 + 25.410 + 1.767 = 53.75 \text{ ksi} \]
\[ \text{or} \]
\[ \frac{53.75(100)}{202.5} = 26.54\% \]

\[ f_{pe} = \text{effective final prestress} = f_{pi} - \Delta f_{pT} = 202.5 - 53.75 = 148.75 \text{ ksi} \]

Check Prestressing stress limit at service limit state:
\[ f_{pe} \leq 0.8f_{py} = 0.8(243) = 194.4 \text{ ksi} > 148.75 \text{ ksi} \quad \text{(O.K.)} \]

Total prestressing force after all losses
\[ P_{pe} = 52(0.153)(148.75) = 1183.46 \text{ Kips} \]
A.2.7.2.7

Final Stresses at midspan

Concrete stress at the top fiber of the beam, $f_i$

Effective prestress and permanent loads, Service I:

$$f_i = \frac{P_{pe}}{A} + \frac{P_{pe} e_c}{S_i} + \frac{M_g + M_S}{S_{ig}} + \frac{M_{SDL}}{S_{ig}}$$

$$= \frac{1183.46}{788.4} - \frac{19.29(1183.46)}{8902.67} + \frac{(1209.98 + 1179.03)(12)}{8902.67} + \frac{347.81(12)}{54083.9}$$

$$= 1.501 - 2.564 + 3.220 + 0.077 = 2.234 \text{ ksi}$$

Allowable compression stress limit for this load combination = $0.45f'_c$

$$f'_c \text{ reqd.} = \frac{2234}{0.45} = 4964.44 \text{ psi}$$

1/2(Effective prestress + permanent loads) + transient loads, Service I:

$$f_i = 0.5 \left( \frac{P_{pe}}{A} + \frac{P_{pe} e_c}{S_i} + \frac{M_g + M_S}{S_{ig}} + \frac{M_{SDL}}{S_{ig}} + \frac{(M_{LT} + M_{LL})}{S_{ig}} \right)$$

$$= 0.5(2.234) + \frac{(1423 + 602.72)(12)}{54083.9} = 1.117 + 0.449 = 1.566 \text{ ksi}$$

Allowable compression stress limit for this load combinations = $0.4f'_c$

$$f'_c \text{ reqd.} = \frac{1566}{0.4} = 3915 \text{ psi}$$

Effective prestress + permanent and transient loads, Service I:

$$f_i = \frac{P_{pe}}{A} + \frac{P_{pe} e_c}{S_i} + \frac{M_g + M_S}{S_{ig}} + \frac{M_{SDL}}{S_{ig}} + \frac{(M_{LT} + M_{LL})}{S_{ig}}$$

$$= 2.234 + \frac{(1423 + 602.72)(12)}{54083.9} = 2.234 + 0.449 = 2.683 \text{ ksi}$$

Allowable compression stress limit for this load combinations = $0.6f'_c$

$$f'_c \text{ reqd.} = \frac{2683}{0.6} = 4471.67 \text{ psi}$$

Bottom fiber stress in concrete at midspan, Service III

$$f_{bf} = \frac{P_{pe}}{A} + \frac{P_{pe} e_c}{S_b} - f_b$$

$$f_{bf} = \frac{1183.46}{788.4} + \frac{19.29(1183.46)}{10521.33} - 4.126 = 1.501 + 2.170 - 4.126 = -0.455 \text{ ksi}$$

Allowable tension in concrete = $0.19\sqrt{f'_c}$

$$f'_c \text{ reqd.} = 1000\left( \frac{0.455}{0.19} \right)^2 = 5734.76 \text{ psi} \quad \text{(controls)}$$
A.2.7.2.8 Initial Stresses at Hold down point

Initial concrete stress at top fiber of the beam at hold down point

\[ f_{ti} = \frac{P_t}{A} + \frac{P_i c_i}{S_t} + \frac{M_g}{S_t} \]

where \( M_g \) = moment due to beam self weight at hold down point = 1197.87 K-ft.

\[ f_{ti} = \frac{1456.35}{788.4} + \frac{19.29(1456.35)}{8902.67} + \frac{1197.87(12)}{8902.67} = 1.847 - 3.155 + 1.615 = 0.307 \text{ ksi} \]

\[ f_{bi} = \frac{P_t}{A} + \frac{P_i c_i}{S_b} - \frac{M_g}{S_b} \]

\[ f_{bi} = \frac{1456.35}{788.4} + \frac{19.29(1456.35)}{10521.33} + \frac{1197.87(12)}{10521.33} = 1.847 + 2.670 - 1.366 = 3.151 \text{ ksi} \]

Compression stress limit at hold down point = 0.6\( f'_{ci} \)

\[ f'_{ci \text{ reqd.}} = \frac{3151}{0.6} = 5251.67 \text{ psi} \]

A.2.7.2.9 Initial Stresses at End

Assuming 10 web strands are draped to top location (5 rows)

\[ c_i = 24.75 - \frac{10(2+4+6) + 8(8) + 4(10) + 2(52 + 50 + 48 + 46 + 44)}{52} = 11.21 \text{ in.} \]

\[ f_{ti} = \frac{P_t}{A} + \frac{P_i c_i}{S_t} + \frac{M_g}{S_t} \]

\[ f_{ti} = \frac{1456.35}{788.4} - \frac{11.21(1456.35)}{8902.67} = 1.847 - 1.834 = 0.013 \text{ ksi} \]

\[ f_{bi} = \frac{P_t}{A} + \frac{P_i c_i}{S_b} \]

\[ f_{bi} = \frac{1456.35}{788.4} + \frac{11.21(1456.35)}{10521.33} = 1.847 + 1.552 = 3.399 \text{ ksi} \]

Compression stress limit = 0.6\( f'_{ci} \)

\[ f'_{ci \text{ reqd.}} = \frac{3399}{0.6} = 5665 \text{ psi} \] (controls)
A.2.7.3 Iteration 3

A.2.7.3.1 Elastic Shortening

Number of strands = 52

\[
\Delta f_{ES} = \frac{E_p}{E_{ci}} f_{egp}
\]

where

\[
f_{egp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_e) e_c}{I}
\]

where

\[P_i = \text{pretension force after allowing for the initial losses, assuming 9.61\% initial losses} = (\text{number of strands})(\text{area of each strand})(0.9039(f_{pi})) = 52(0.153)(0.9039)(202.5) = 1456.26 \text{ Kips}\]

\[M_e = \text{Unfactored bending moment due to beam self weight} = 1209.98 \text{ K-ft.}\]

\[e_c = \text{eccentricity of the strand at the midspan} = 19.29 \text{ in.}\]

\[
f_{egp} = \frac{1456.26}{788.4} + \frac{1456.26(19.29)^2}{260403} - \frac{1209.98(12)(19.29)}{260403} = 1.847 + 2.081 - 1.076 = 2.852 \text{ ksi}
\]

\[E_{ci} = \text{modulus of elasticity of beam at release} = (w_c)^{1.5}(33)\sqrt{f_{ci}}\]

where

\[w_c = \text{weight of concrete} = 150 \text{ pcf}\]

\[f_{ci} = 5665 \psi i\]

\[
E_{ci} = (150)^{1.5}(33)\sqrt{5665} \frac{1}{1000} = 4563 \text{ ksi}
\]

\[E_p = \text{modulus of elasticity of prestressing reinforcement} = 28500 \text{ ksi}\]

Therefore loss due to elastic shortening:

\[
\Delta f_{ES} = \frac{28500}{4563} (2.852) = 17.81 \text{ ksi}
\]

A.2.7.3.2 Shrinkage

\[
\Delta f_{SR} = 17 - 0.15 H
\]

where \(H\) is relative humidity = 60%

\[
\Delta f_{SR} = [17 - 0.15(60)] = 8.0 \text{ ksi}
\]
A.2.7.3.3 Creep of Concrete

\[ \Delta f_{pCR} = 12f_{cp} - 7\Delta f_{cdp} \]  
[LRFD Eq. 5.9.4.3-1]

where

\[ \Delta f_{cdp} = \text{change of stresses at the center of gravity of the prestressing steel due to permanent loads except the dead load present at the time the prestress force is applied calculated at the same section as } f_{cp} \]

\[ \Delta f_{cdp} = \frac{M_S e_c}{I} + \frac{M_{SDL} (y_{bc} - y_{bs})}{I_c} \]

where

\[ M_S = \text{slab moment} = 1179.03 \text{ K-ft.} \]
\[ M_{SDL} = \text{superimposed dead load moment} = 347.81 \text{ K-ft.} \]
\[ y_{bc} = 41.157 \text{ in.} \]
\[ y_{bs} = \text{the distance from center of gravity of the strand at midspan to the bottom of the beam} = 24.75 - 19.29 = 5.46 \text{ in.} \]
\[ I = \text{moment of inertia of the non-composite section} = 260403 \text{ in.}^4 \]
\[ I_c = \text{moment of inertia of composite section} = 694599.5 \text{ in.}^4 \]

\[ \Delta f_{cdp} = \frac{1179.03(12)(19.29)}{260403} + \frac{(347.81)(12)(41.157 - 5.46)}{694599.5} \]

\[ \Delta f_{cdp} = 1.048 + 0.214 = 1.262 \text{ ksi} \]

\[ \Delta f_{pCR} = 12(2.852) - 7(1.262) = 25.39 \text{ ksi}. \]

A.2.7.3.4 Relaxation of Prestressing Strands

A.2.7.3.4.1 Relaxation before Transfer

Loss due to Relaxation of steel before transfer

\[ \Delta f_{pR1} = 0 \]

A.2.7.3.4.2 Relaxation after Transfer

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

\[ \Delta f_{pRS} = 30\% \left[ 20.0 - 0.4 \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR}) \right] \]
\[ = 0.3[20.0 - 0.4(17.81) - 0.2(8 + 25.39)] = 1.859 \text{ ksi} \]

TxDOT Bridge Design Manual (Pg. # 7-85) recommends that 50% of the final steel relaxation loss be considered for calculation of total initial loss given as [Elastic shortening loss + 0.50(total steel relaxation loss)]

\[ \text{Initial Prestress loss } % = \frac{(\Delta f_{pES} + 0.5\Delta f_{pR2})100}{f_p} \]
Therefore next trial is required assuming 9.25% initial losses

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$  
[LRFD Art. 5.9.5.2.3a]

where

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

$P_i =$ pretension force after allowing for the initial losses, assuming 9.25% initial losses = (number of strands)(area of each strand)(0.9075$f_{pi}$)

$$52(0.153)(0.9075)(202.5) = 1462.06 \text{ Kips}$$

$M_g = 1209.98 \text{ K-ft.}$

e_c = 19.29 \text{ in.}$

$$\begin{align*}
    f_{cgp} &= \frac{1462.06}{788.4} + \frac{1462.06(19.29)}{260403} - \frac{1209.98(12)(19.29)}{260403} \\
    &= 1.854 + 2.089 - 1.076 = 2.867 \text{ ksi}
\end{align*}$$

$E_{ci} = 4563 \text{ ksi}$

$E_p = 28500 \text{ ksi}$

Therefore loss due to elastic shortening:

$$\Delta f_{pES} = \frac{28500}{4563}(2.867) = 17.91 \text{ ksi}$$

$$\Delta f_{pSR} = 8.0 \text{ ksi}$$

For pretensioned members with 270 ksi low-relaxation strands, loss due to relaxation after transfer

$$\Delta f_{pR2} = 30\% \ [20.0 - 0.4(\Delta f_{pES}) - 0.2(\Delta f_{pSR} + \Delta f_{pCR})]$$

$$\Delta f_{pR2} = 30\% \ [20.0 - 0.4(17.91) - 0.2(8 + 25.57)] = 1.837 \text{ ksi}$$

Initial Prestress loss % = \frac{\left(\Delta f_{pES} + 0.5\Delta f_{pR2}\right)100}{f_{pi}}

$$= \frac{[17.91 + 0.5(1.837)]100}{202.5} = 9.30\% \approx 9.25 \% \text{ (assumed initial prestress losses)}$$
A.2.7.3.5  
Total Losses at Transfer

Total Initial loss, $\Delta f_{pi} = (\Delta f_{pES} + 0.5\Delta f_{pR2}) = [17.91 + 0.5(1.837)] = 18.83$ ksi  

$P_i = \text{effective pretension force after allowing for the initial losses}$  

$= 52(0.153)(183.67) = 1461.28$ Kips  

A.2.7.3.6  
Total Losses at Service Loads

$\Delta f_{pES} = 17.910$ ksi  
$\Delta f_{pSR} = 8.000$ ksi  
$\Delta f_{pCR} = 25.570$ ksi  
$\Delta f_{pR2} = 1.837$ ksi  

Total final loss, $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2}$  

$= 17.910 + 8.000 + 25.570 + 1.837 = 53.32$ ksi  

or $\frac{53.32(100)}{202.5} = 26.33\%$  

$f_{pe} = \text{effective final prestress} = f_{pi} - \Delta f_{pT} = 202.5 - 53.32 = 149.18$ ksi  

Check Prestressing stress limit at service limit state:  

$f_{pe} \leq 0.8f_{yp} = 0.8(243) = 194.4$ ksi $> 149.18$ ksi  

(O.K.)  

Total prestressing force after all losses  

$P_{pe} = 52(0.153)(149.18) = 1186.88$ Kips  

A.2.7.3.7  
Final Stresses at midspan

Concrete stress at the top fiber of the beam, $f_t$  

Effective prestress + permanent loads, Service I:  

$$f_t = \frac{P_{pe}e_c + M_g + M_s + M_{SDL}}{S_t}$$  

$$= \frac{1186.88 + (1209.98 + 1179.03)(12)}{8902.67} + \frac{347.81(12)}{54083.9}$$  

$$= 1.505 - 2.572 + 3.22 + 0.077 = 2.23\text{ ksi}$$  

Allowable compression stress limit for this load combination = 0.45$f'c$  

$f'c\text{ reqd.} = 2230/0.45 = 4955.55$ psi

$\frac{1}{2}(\text{Effective prestress + permanent loads}) + \text{transient loads}, \text{ Service I:}$  

$$f_t = 0.5 \left( \frac{P_{pe}e_c}{S_t} + \frac{M_g + M_s + M_{SDL}}{S_t} \right) + \frac{(M_{LT} + M_{LL})}{S_t}$$  

$$= 0.5(2.230) + \frac{(1423 + 602.72)(12)}{54083.9} = 1.115 + 0.449 = 1.564\text{ ksi}$$

Detailed Design Examples - 32
Allowable compression stress limit for this load combinations = 0.4\( f'_c \)
\[ f'_c \text{ reqd.} = \frac{1564}{0.4} = 3910 \text{ psi} \]

Effective prestress + permanent loads + transient loads, Service I:
\[
f_i = \frac{P_{pe} + P_{pe} c_x}{A} + \frac{M_g + M_s}{S_t} + \frac{M_{SDL}}{S_{tg}} + \frac{(M_{LT} + M_{LL})}{S_{tg}}
\]
\[
= 2.230 + \frac{(1423 + 602.72)12}{54083.9} = 2.230 + 0.449 = 2.679 \text{ ksi}
\]

Allowable compression stress limit for this load combinations = 0.6\( f'_c \)
\[ f'_c \text{ reqd.} = \frac{2679}{0.6} = 4465 \text{ psi} \]

Bottom fiber stress in concrete at midspan, Service III
\[
f_{bf} = \frac{P_{pe} + P_{pe} c_x}{A} + \frac{M_g}{S_b} - f_b
\]
\[
= 1186.88 + \frac{19.29(1186.88)}{10521.33} - 4.126 = 1.505 + 2.176 - 4.126 = -0.445 \text{ ksi}
\]

Allowable tension in concrete = 0.19\( \sqrt{f'_c} \)
\[ f'_c \text{ reqd.} = 1000 \left( \frac{0.445}{0.19} \right)^2 = 5485.46 \text{ psi} \quad \text{(controls)}
\]

Initial concrete stress at top fiber of the beam at hold down point
\[
f_i = \frac{P_i + P_i c_x}{A} + \frac{M_g}{S_t}
\]
where \( M_g \) = moment due to beam self weight at hold down point = 1197.87 K-ft.

\[
f_i = \frac{1461.28}{788.4} - \frac{19.29(1461.28)}{8902.67} + \frac{1197.87(12)}{8902.67} = 1.853 - 3.166 + 1.615 = 0.302 \text{ ksi}
\]

Compression stress limit at hold down point = 0.6\( f'_{ci} \)
\[ f'_{ci} \text{ reqd.} = \frac{3166}{0.6} = 5276.67 \text{ psi} \]
A.2.7.3.9

Initial Stresses at End

Assuming 10 web strands are draped to top location (5 rows)

\[ e_c = 24.75 - \frac{10(2+4+6) + 8(8) + 4(10) + 2(52 + 50 + 48 + 46 + 44)}{52} = 11.21 \text{ in.} \]

\[ f_{lt} = \frac{P_i}{A} \frac{\epsilon c}{S_t} \]

\[ f_{lt} = \frac{1461.28}{788.4} - \frac{11.21(1461.28)}{8902.67} = 1.853 - 1.840 = 0.013 \text{ ksi} \]

Concrete stresses at bottom fiber

\[ f_{bi} = \frac{P_i}{A} + \frac{\epsilon c}{S_b} \]

\[ f_{bi} = \frac{1461.28}{788.4} + \frac{11.21(1461.28)}{10521.33} = 1.853 + 1.557 = 3.410 \text{ ksi} \]

Compression stress limit = 0.6f'ci

\[ f'_{ci} \text{ reqd.} = \frac{3410}{0.6} = 5683.33 \text{ psi} \quad \text{(controls)} \]

Therefore provide \( f'_{ci} = 5683.33 \text{ psi} \)

\( f''_c = 5683.33 \text{ psi (} f''_c \text{ should be at least equal to } f'_{ci}) \)

52 – ½ in. diameter, 10 harped at the end, GR 270 low relaxation strands
No. of Strands | Distance from bottom (in.) | No. of Strands | Distance from bottom (in.)
---|---|---|---
6 | 10 | 2 | 52
10 | 8 | 2 | 50
12 | 6 | 2 | 48
12 | 4 | 2 | 46
12 | 2 | 2 | 44

**Fig. A.2.7.1**
Final Strand Pattern at Midspan

**Fig. A.2.7.2**
Final Strand Pattern at Ends

**Fig. A.2.7.3** Longitudinal strand profile (Half of the beam length is shown)

Note: The hold down distance of 49.4 ft. is measured from the end of the beam.
The distance between the center of gravity of the 10 harped strands and the top of the beam at the end of the beam is \( \frac{2(2) + 2(4) + 2(6) + 2(8) + 2(10)}{10} = 6 \) in.

The distance between the center of gravity of the 10 harped strands and the bottom of the beam at harp points is \( \frac{2(2) + 2(4) + 2(6) + 2(8) + 2(10)}{10} = 6 \) in.

The distance between the center of gravity of the 10 harped strands and the top of the beam at transfer length section is \( 6 \) in. + \( \frac{(54 \text{ in} - 6 \text{ in} - 6 \text{ in})}{49.4 \text{ ft.}} \times (2.5 \text{ ft.}) = 8.12 \) in.

The distance between the center of gravity of the 42 straight strands and the bottom of the beam at all locations is \( \frac{10(2) + 10(4) + 10(6) + 8(8) + 4(10)}{42} = 5.33 \) in.

### A.2.8 CHECK FOR DISTRIBUTION FACTORS

#### A.2.8.1 Distribution factor for Bending Moment

For all limit states except fatigue limit state:

For two or more lanes loaded:

\[
DFM = 0.075 + \left( \frac{S}{9.5} \right)^{0.6} \left( \frac{S}{L} \right)^{0.2} \left( \frac{K_g}{12.0Lt_c} \right)^{0.1} \quad [\text{LRFD Table 4.6.2.2.2b-1}]
\]

Provided that:

- \( 3.5 \leq S \leq 16 \); \( S = 8.0 \) ft (O.K.)
- \( 4.5 \leq t_c \leq 12 \); \( t_c = 8.0 \) in (O.K.)
- \( 20 \leq L \leq 240 \); \( L = 108.583 \) ft. (O.K.)
- \( N_b \geq 4 \); \( N_b = 6 \) (O.K.)
- \( 10,000 \leq K_g \leq 7,000,000 \) (O.K., as shown below)

Where

- \( DFM = \) distribution factor for moment for interior beam
- \( S = \) spacing of beams = 8.0 ft
- \( L = \) beam span = 108.583 ft
- \( t_c = \) depth of concrete deck = 8.0 in.
- \( K_g = \) longitudinal stiffness parameter, in.\(^4\)
\[ K_g = n \left( I + A e_g^2 \right) \]  
\[ \text{where} \]
\[ A \text{ = cross-sectional area of the beam (non-composite section)} \]
\[ A = 788.4 \text{ in.}^2 \]
\[ I \text{ = moment of inertia of beam (non-composite section)} = 260,403.0 \text{ in.}^4 \]
\[ e_g \text{ = distance between centers of gravity of the beam and slab, in.} \]
\[ = \left( \frac{t_b}{2} + y_t \right) = \left( \frac{8}{2} + 29.25 \right) = 33.25 \text{ in.} \]
\[ n \text{ = modular ratio between beam and slab materials} = \frac{E_c(\text{beam})}{E_c(\text{deck})} \]
\[ E_c(\text{beam}) = \text{modulus of elasticity of beam at service} = (w_c)^{1.5}(33)\sqrt{f'c} \]
\[ \text{where} \]
\[ w_c = \text{weight of beam concrete} = 150 \text{ pcf} \]
\[ f'c = 5683.33 \text{ psi} \]
\[ E_c(\text{beam}) = (150)^{1.5}(33)\sqrt{5683.33} \frac{1}{1000} = 4570.38 \text{ ksi} \]
\[ E_c(\text{deck}) = \text{modulus of elasticity of slab} = (w_c)^{1.5}(33)\sqrt{f'c} \]
\[ \text{where} \]
\[ w_c = \text{weight of slab concrete} = 150 \text{ pcf} \]
\[ f'c = 4000.0 \text{ psi} \]
\[ E_c(\text{deck}) = (150)^{1.5}(33)\sqrt{4000} \frac{1}{1000} = 3834.25 \text{ ksi} \]
\[ n = \frac{4570.38}{3834.25} = 1.192 \]

Therefore

\[ K_g = 1.192[260403 + 788.4(33.25)^2] = 1,349,377.94 \text{ in.}^4 \]

\[ \text{DFM} = 0.075 + \left( \frac{8}{9.5} \right)^{0.6} \left( \frac{8}{108.583} \right)^{0.2} \left( \frac{1349377.94}{12.0(108.583)(8)} \right)^{0.1} \]

\[ = 0.075 + (0.902)(0.593)(1.073) = 0.649 \text{ lanes/beam} \]

For one design lane loaded:

\[ \text{DFM} = 0.06 + \left( \frac{S}{14} \right)^{0.4} \left( \frac{S}{L} \right)^{0.3} \left( \frac{K_g}{12.0L^2} \right)^{0.1} \]
AASHTO Type IV - LRFD Specifications

**Concrete Stresses at Transfer**

A.2.8.1.1

**Allowable Stress limits**

[LRFD Art. 5.9.4]

**Compression:** $0.6 f_{ci} = 0.6(5683.33) = +3410.0$ psi = 3.410 ksi (compression)

**Tension:**

Without bonded reinforcement

$0.0948 \sqrt{f_{ci}} \leq 0.200$ ksi; $-0.0948 \sqrt{5.683} = -0.226$ ksi

Therefore $-0.200$ ksi controls

With bonded auxiliary reinforcement which is sufficient to resist 120% of the tension force in the cracked concrete

$-0.24 \sqrt{f_{ci}} = -0.24 \sqrt{5.683} = -0.572$ ksi

---

Thus, the case of two or more lanes loaded controls

DFM = 0.649 lanes/beam

For fatigue limit state:

LRFD Specifications, Art. 3.4.1, states that for fatigue limit state, a single design truck should be used. However, live load distribution factors given in LRFD Article 4.6.2.2 take into consideration the multiple presence factor, $m$. LRFD Article 3.6.1.1.2 states that the multiple presence factor, $m$, for one design lane loaded is 1.2. Therefore, the distribution factor for one design lane loaded with the multiple presence factor removed should be used. The distribution factor for the fatigue limit state is: $0.452/1.2 = 0.377$ lanes/beam.

Percent change in DFM = $\frac{0.649 - 0.639}{0.649} \times 100 = 1.54\%$

Percent change in DFM for fatigue limit state = $\frac{0.377 - 0.371}{0.377} \times 100 = 1.6\%$

The change in the distribution factors is very less and can be neglected as they won’t impact the moments by a great amount. The moments obtained using previous distribution factors are used in the following design.
A.2.8.1.2 Stresses at Beam End

Total prestressing force after transfer = 52(0.153)(183.67) = 1461.28 kips.

Stresses at end are checked only at release, because it almost always governs.

Assuming all web strands are draped to top location (5 rows)

Eccentricity of strands at the end of the beam

\[ e_e = 24.75 - \frac{10(2+4+6) + 8(8) + 4(10) + 2(52 + 50 + 48 + 46 + 44)}{52} = 11.21 \text{ in.} \]

End concrete stress at top fiber at release

\[ f_{ti} = \frac{P_i - P_i e_e}{A} \]

\[ f_{ti} = \frac{1461.28 - 11.21(1461.28)}{788.4} = \frac{1461.28}{8902.67} = 1.853 - 1.84 = 0.013 \text{ ksi} \]

Allowable compression: +3.410 ksi >> 0.013 ksi (reqd.) (O.K.)

There is no need for additional bonded reinforcement as there is no tension in the top fiber of the beam.

End concrete stress at bottom fiber at release

\[ f_{bi} = \frac{P_i + P_i e_e}{A} \]

\[ f_{bi} = \frac{1461.28 + 11.21(1461.28)}{788.4} = \frac{1461.28}{10521.33} = 1.853 + 1.557 = 3.410 \text{ ksi} \]

Allowable compression: +3.410 ksi = 3.410 ksi (reqd.) (O.K.)

A.2.8.1.3 Stresses at Transfer Length Section

Stresses at transfer length are checked only at release, because it almost always governs.

Transfer length = 60(strand diameter) \[ \text{[LRFD Art. 5.8.2.3]} \]

= 60(0.50) = 30 in. = 2.5 ft.

Transfer length section is located at a distance of 2.5 ft. from end of the beam or at a point 1.96 ft. from center of the bearing as the beam extends 6.5 in. beyond the bearing centerline. Overall beam length of 109.67 ft. is considered for the calculation of bending moment at transfer length.

Moment due to beam self weight, \( M_g = 0.5wx(L-x) \)

\[ M_g = 0.5(0.821)(2.5)(109.67 - 2.5) \]

= 110.00 K –ft.
Concrete stress at top fiber of the beam

\[ f_t = \frac{P_t}{A} \frac{P_t \epsilon_t}{S_t} + \frac{M_g}{S_t} \]

Strand eccentricity at transfer section, \( e_t = \epsilon_c - (\epsilon_c - \epsilon_e) \frac{(48.862 - x)}{48.862} \)

\[ e_t = 19.29 - (19.29 - 11.21) \frac{(48.862 - 2.5)}{48.862} = 11.62 \text{ in.} \]

\[ f_t = \frac{1461.28}{788.4} \frac{11.62(1461.28)}{8902.67} + \frac{110.00(12)}{8902.67} - 1.853 - 1.907 + 0.148 = 0.094 \text{ ksi} \]

Allowable compression: +3.410 ksi > 0.094 ksi (reqd.) (O.K.)

Concrete stress at the bottom fiber of the beam

\[ f_b = \frac{P_t}{A} + \frac{P_t \epsilon_c}{S_b} - \frac{M_g}{S_b} \]

\[ f_{b1} = \frac{1461.28}{788.4} + \frac{11.62(1461.28)}{10521.33} - \frac{110.00(12)}{10521.33} = 1.853 + 1.614 - 0.125 = 3.342 \text{ ksi} \]

Allowable compression: +3.410 ksi > 3.342 ksi (reqd.) (O.K.)

A.2.8.1.4 Stresses at Harp points

Eccentricity of the strands at harp points is same as at midspan

\( \epsilon_{harp} = \epsilon_c = 19.29 \text{ in.} \)

Distance of harp point from bearing center line = 48.862 ft.

Distance of harp point from beam end = (6.5/12) + 48.862 = 49.4 ft.

Bending moment at harp point due to beam self weight based on overall beam length, \( M_g = 0.5wx(L-x) = 0.5(0.821)(49.4)(109.67 - 49.4) = 1222.2 \text{ K-ft.} \)

Concrete stress at top fiber of the beam at hold down point

\[ f_t = \frac{P_t}{A} - \frac{P_t \epsilon_c}{S_t} + \frac{M_g}{S_t} \]

\[ f_t = \frac{1461.28}{788.4} - \frac{19.29(1461.28)}{8902.67} + \frac{1222.2(12)}{8902.67} = 1.853 - 3.166 + 1.647 = 0.334 \text{ ksi} \]

Allowable compression: +3.410 ksi >> 0.334 ksi (Reqd.) (O.K.)

Concrete stresses in bottom fibers of the beam at hold down point

\[ f_b = \frac{P_t}{A} + \frac{P_t \epsilon_c}{S_b} - \frac{M_g}{S_b} \]
**AASHTO Type IV - LRFD Specifications**

**A.2.8.1.5 Stresses at Midspan**

Bending moment at midspan due to beam self-weight based on overall length

\[ M_b = 0.5wx (L-x) \]

\[ = 0.5(0.821)(54.835)(109.67 - 54.835) = 1234.32 \text{ K-ft.} \]

Concrete stress at top fiber of the beam at midspan

\[ f_t = \frac{P_{si}}{A} - \frac{P_{si} \cdot e_c}{S_t} + \frac{M_d}{S_t} \]

\[ f_t = \frac{1461.28}{788.4} - \frac{19.29(1461.28)}{8902.67} + \frac{1234.32(12)}{8902.67} = 1.853 - 3.166 + 1.664 = 0.351 \text{ ksi} \]

Allowable compression: +3.410 ksi > 0.351 ksi (Reqd.) (O.K.)

Concrete stresses in bottom fibers of the beam at hold down point

\[ f_b = \frac{P_{si}}{A} + \frac{P_{si} \cdot e_c}{S_b} - \frac{M_d}{S_b} \]

\[ f_b = \frac{1461.28}{788.4} + \frac{19.29(1461.28)}{10521.33} - \frac{1234.32(12)}{10521.33} = 1.853 + 2.679 - 1.408 = 3.124 \text{ ksi} \]

Allowable compression: +3.410 ksi > 3.124 ksi (reqd.) (O.K.)

**A.2.8.1.6 Stress Summary at transfer**

<table>
<thead>
<tr>
<th></th>
<th>Top of beam</th>
<th>Bottom of beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>At End</td>
<td>+0.013</td>
<td>+3.410</td>
</tr>
<tr>
<td>At transfer length section</td>
<td>+0.094</td>
<td>+3.342</td>
</tr>
<tr>
<td>At harp Points</td>
<td>+0.334</td>
<td>+3.138</td>
</tr>
<tr>
<td>At Midspan</td>
<td>+0.351</td>
<td>+3.124</td>
</tr>
</tbody>
</table>

[LRFD Art. 5.9.4.2]

**Concrete Stresses at Service Loads**

**A.2.8.2.1 Allowable Stress Limits**

**Compression**

Due to (Effective prestress + permanent loads) for load combination service I

\[ 0.45f'_c = 0.45(5683.33)/1000 = +2.557 \text{ ksi (for precast beam)} \]

\[ 0.45f'_c = 0.45(4000)/1000 = +1.800 \text{ ksi (for slab)} \]

Detailed Design Examples - 41
Due to $\frac{1}{2}$ (effective prestress + permanent loads) + transient loads for load combination Service I

$$0.40f_c = 0.40(5683.33)/1000 = +2.273 \text{ ksi (for precast beam)}$$

Due to permanent and transient loads for load combination Service I

$$0.60f_c = 0.60(5683.33)/1000 = +3.410 \text{ ksi (for precast beam)}$$

$$0.60f_c = 0.60(4000)/1000 = +2.400 \text{ ksi (for slab)}$$

Tension:

For components with bonded prestressing tendons

- for load combination Service III: $-0.19 \sqrt{f_c}$
- for precast beam: $-0.19 \sqrt{5.683} = -0.453 \text{ ksi}$

Concrete stresses at top fiber of the beam:

The compressive stresses are checked for two cases

1. Effective prestress + permanent loads, Service I

$$f_t = \frac{P_{pe}}{A} - \frac{P_{pe} f_c e_c}{S_t} + \frac{M_g + M_s}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$f_t = \frac{1186.88}{788.4} - \frac{19.29(1186.88)}{8902.67} + \frac{(1209.98 + 1179.03)(12)}{8902.67} + \frac{(347.81)(12)}{54083.9}$$

$$f_t = 1.505 - 2.572 + 3.220 + 0.077 = 2.230 \text{ ksi}$$

Allowable compression: $+2.557 \text{ ksi} > +2.230 \text{ ksi (reqd.) (O.K.)}$

2. $\frac{1}{2}$ (Effective prestress + permanent loads) + transient loads, Service I:

$$f_t = 0.5 \left( \frac{P_{pe}}{A} - \frac{P_{pe} f_c e_c}{S_t} + \frac{M_g + M_s}{S_t} + \frac{M_{SDL}}{S_{tg}} + \frac{M_{LL} + M_{LT}}{S_{tg}} \right)$$

$$0.5 \left( \frac{1186.88}{788.4} - \frac{19.29(1186.88)}{8902.67} + \frac{(1209.98 + 1179.03)(12)}{8902.67} + \frac{(347.81)(12)}{54083.9} + \frac{(1423.00 + 602.72)(12)}{54083.9} \right)$$

$$f_t = 0.5(1.505 - 2.572 + 3.220 + 0.077) + 0.449 = 1.564 \text{ ksi}$$

Allowable compression: $+2.273 \text{ ksi} > +1.564 \text{ ksi (reqd.) (O.K.)}$
3. Under permanent and transient loads, Service I:

\[ f_t = \frac{P_{pe}}{A} - \frac{P_{pe} \, e_c}{S_t} + \frac{M_g + M_S}{S_{tg}} + \frac{M_{SDL} + M_{LL} + M_{LT}}{S_{tg}} \]

\[
\begin{align*}
1186.88 & \quad 19.29(1186.88) \\
788.4 & \quad (1209.98 + 1179.03)(12) \\
& \quad 347.81(12) \\
& \quad (1423.00 + 602.72)(12) \\
& \quad 12 - + + \\
\end{align*}
\]

\[ f_t = 1.505 - 2.572 + 3.220 + 0.077 + 0.449 = 2.679 \text{ ksi} \]

Allowable compression: +3.410 ksi > +2.679 ksi (reqd.) (O.K.)

Stresses at the top of the deck

1. Under permanent loads, Service I

\[ f_t = \frac{M_{SDL}}{S_{tc}} = \frac{(347.81)(12)}{33325.31} = 0.125 \text{ ksi} \]

Allowable compression: +1.800 ksi > +0.125 ksi (reqd.) (O.K.)

2. Under permanent and transient loads, Service I

\[ f_{tc} = \frac{M_{SDL} + M_{LL} + M_{LT}}{S_{tc}} = \frac{(347.81 + 1423.00 + 602.72)(12)}{33325.31} = +0.855 \text{ ksi} \]

Allowable compression: +2.400 ksi > +0.855 ksi (reqd.) (O.K.)

Concrete stresses at bottom fiber of the beam, Service III:

\[ f_b = \frac{P_{pe}}{A} + \frac{P_{pe} \, e_c}{S_b} - \frac{M_g + M_S}{S_{sb}} - \frac{M_{SDL} + 0.8(M_{LL} + M_{LT})}{S_{sb}} \]

\[
\begin{align*}
1186.88 & \quad 19.29(1186.88) \\
788.4 & \quad (1209.98 + 1179.03)(12) \\
& \quad 347.81 + 0.8(1423.00 + 602.72)(12) \\
& \quad 12 - - + \\
\end{align*}
\]

\[ f_b = 1.505 + 2.176 - 2.725 - 1.400 = -0.444 \text{ ksi} \]

Allowable Tension: –0.453 ksi (O.K.)

In regions of compressive stress due to permanent loads and prestress, fatigue is considered only if the compressive stress is less than twice the maximum tensile live load stress resulting from fatigue load combination [LRFD Art. 5.5.3.1]

At midspan, bottom fiber stress due to permanent loads and prestress is:

\[ \frac{P_{pe}}{A} + \frac{P_{pe} \, e_c}{S_b} - \frac{M_g + M_S}{S_{sb}} - \frac{M_{SDL}}{S_{sb}} \]
A.2.8.2.4
Summary of Stresses at Service loads

\[
\frac{1186.88}{788.4} + \frac{19.29(1186.88)}{10521.33} - \frac{(1209.98+1179.03)(12)}{10521.33} - \frac{347.81(12)}{16876.83} = 1.505 + 2.176 - 2.725 - 0.247 = 0.709 \text{ ksi}
\]

Stress at the bottom fiber of the beam due to fatigue load combination is:

\[
\frac{0.75(M_f)}{S_{bc}} = \frac{0.75(605.16)(12)}{16876.83} = -0.323 \text{ ksi}
\]

\[2(0.323 \text{ ksi}) = 0.646 \text{ ksi} < 0.709 \text{ ksi}, \text{ therefore fatigue check need not be performed.}\]
A.2.9 STRENGTH LIMIT STATE

Total ultimate bending moment for Strength I is:
\[ M_u = 1.25(DC) + 1.5(DW) + 1.75(LL + IM) \]

where
- DC = bending moment due to dead loads except wearing surface
- DW = bending moment due to wearing surface load
- LL + IM = bending moment due to live load and impact

The ultimate bending moment at midspan is
\[ M_u = 1.25(1209.98 + 1179.03 + 162.12) + 1.5(185.70) + 1.75(1423 + 602.72) \]
\[ = 7012.47 \text{ Kip-ft.} \]

\[ f_{pe} = 149.18 \text{ ksi} > 0.5f_{pu} = 0.5(270) = 135 \text{ ksi} \]

Average stress in pretensioning steel when \( f_{pe} \geq 0.5f_{pu} \):
\[ f_{ps} = f_{pu} \left(1 - \frac{k}{d_p}\right) \]  
\[
\begin{align*}
\text{[LRFD Eq. 5.7.3.1.1-1]} \\
\text{where}
\end{align*}
\[ f_{ps} = \text{average stress in prestressing steel} \\
f_{pu} = \text{specified tensile strength of prestressing steel} = 270 \text{ ksi} \\
k = 2 \left(1.04 - \frac{f_{py}}{f_{pu}}\right) \]  
\[
\begin{align*}
\text{[LRFD Eq. 5.7.3.1.1-2]} \\
\text{= 0.28 for low-relaxation strand} \quad \text{[LRFD Table C5.7.3.1.1-1]} \\
d_p = \text{distance from extreme compression fiber to the centroid of the prestressing tendons} = h - y_{bs} = 62 - 5.46 = 56.54 \text{ in.} \\
c = \text{distance between neutral axis and the compressive face, in.} \\
\end{align*}
\]

To compute \( c \), assume rectangular section behavior and check if the depth of the equivalent compression stress block, \( a \), is less than or equal to \( t_s \)  
\[
\text{[LRFD C5.7.3.2.2]} \\
\text{where} \quad a = \beta_1 c \\
c = \frac{A_{ps}f_{pu} + A_tf_y - A_{t}f'_y}{0.85f'_y b + kA_{ps}} 
\]
\[
\begin{align*}
\text{[LRFD Eq. 5.7.3.1.1.-4]} \\
\text{where} \\
A_{ps} = \text{area of prestressing steel} = 52(0.153) = 7.956 \text{ in.}^2 \\
A_t = \text{area of mild steel tension reinforcement} = 0 \text{ in.}^2 \\
A_{t} = \text{area of compression reinforcement} = 0 \text{ in.}^2 \\
\end{align*}
\]
f_c’ = compressive strength of deck concrete = 4 ksi
f_y = yield strength of tension reinforcement, ksi
f_y’ = yield strength of compression reinforcement, ksi
β_1 = stress factor of compression block [LRFD Art. 5.7.2.2]
= 0.85 for f_c’ ≤ 4.0 ksi
b = effective width of compression flange = 96 in.

\[ c = \frac{7.956(270) + 0 - 0}{0.85(4.0)(0.85)(96) + 0.28(7.956)} = 7.457 \text{ in.} \leq t_s = 8 \text{ in.} \quad \text{O.K.} \]

\[ a = \beta_1 c = 0.85(7.457) = 6.338 \text{ in.} < t_s = 8 \text{ in.} \]

Therefore, the rectangular section behavior assumption is valid.

The average stress in prestressing steel is:

\[ f_{ps} = 270 \left(1 - 0.28 \frac{7.457}{56.54}\right) = 260.03 \text{ ksi} \]

Nominal flexural resistance:

\[ M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2}\right) \quad \text{[LRFD Eq. 5.7.3.2.2-1]} \]

The above equation is a simplified form of LRFD Equation 5.7.3.2.2-1 because no compression reinforcement or mild tension reinforcement is considered and the section behaves as a rectangular section

\[ M_n = (7.956)(260.03) \left(56.54 - \frac{6.338}{2}\right)/12 = 9201.15 \text{ kip-ft.} \]

Factored flexural resistance:

\[ M_r = \phi M_n \quad \text{[LRFD Eq. 5.7.3.2.1-1]} \]

where

\[ \phi = \text{resistance factor} \quad \text{[LRFD Art. 5.5.4.2.1]} \]

= 1.0 for flexure and tension of prestressed concrete

\[ M_r = 1(9201.15) = 9201.15 \text{ kip-ft.} > M_u = 7012.47 \text{ kip-ft} \quad \text{O.K.} \]

Detailed Design Examples - 46
A.2.10 LIMITS OF REINFORCEMENT

A.2.10.1 Maximum Reinforcement

The amount of prestressed and non-prestressed reinforcement should be such that

\[
\frac{c}{d_e} \leq 0.42
\]

[LRFD Art. 5.7.3.3.1-1]

where

\[
d_e = \frac{A_{ps}f_{ps}d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y}
\]

[LRFD Eq. 5.7.3.3.1-2]

\[A_s = 0, \text{ Therefore } d_e = d_p = 56.54 \text{ in.}\]

\[
\frac{c}{d_e} = \frac{7.457}{56.54} = 0.132 < 0.42 \quad \text{O.K.}
\]

At any section, the amount of prestressed and non-prestressed tensile reinforcement should be adequate to develop a factored flexural resistance, \(M_r\), equal to the lesser of:

1.2 times the cracking strength determined on the basis of elastic stress distribution and the modulus of rupture, and

1.33 times the factored moment required by the applicable strength load combination.

Check at midspan:

Cracking moment:

\[
M_{cr} = (f_r + f_{pe}) S_{bc} - M_{d-nc} \left( \frac{S_{bc}}{S_b} - 1 \right) \leq S_c f_r
\]

[LRFD Eq. 5.7.3.3.2-1]

where

\( f_r = \) modulus of rupture \[ \text{LRFD Art. 5.4.2.6} \]

\( f_r = 0.24 \sqrt{f_c} = 0.24 \sqrt{5.683} = 0.572 \text{ ksi} \)

\( f_{pe} = \) compressive stress in concrete due to effective prestress forces at extreme fiber of section where tensile stress is caused by externally applied loads

\[
f_{pe} = \frac{P_{pe}}{A} + \frac{P_{pe} e_c}{S_b}
\]

[LRFD Art. 5.4.2.6]

where

\[ P_{pe} = \text{effective prestress force after losses} = 1186.88 \text{ Kips}\]

\[ e_c = 19.29 \text{ in.}\]
AASHTO Type IV - LRFD Specifications

A.2.11 SHEAR DESIGN

\[ f_{pe} = \frac{1186.88}{788.4} + \frac{1186.88(19.29)}{10521.33} = 1.505 + 2.176 = 3.681 \text{ ksi} \]

\[ M_{d,nc} = \text{non-composite dead load moment at midspan due to self weight of beam and weight of slab} = 1209.98 + 1179.03 = 2389.01 \text{ kip-ft.} \]

\[ S_b = \text{non-composite section modulus for the extreme fiber of the section where the tensile stress is caused by externally applied loads} = 10521.33 \text{ in.}^4 \]

\[ S_{bc} = \text{composite section modulus for the extreme fiber of the section where the tensile stress is caused by externally applied loads} = 16876.83 \text{ in.}^4 \]

\[ =(0.572 + 3.681)(16876.83) \left( \frac{1}{12} \right) - 2389.01 \left( \frac{16876.83}{10521.33} \right) - 1 \]

\[ = 5981.43 - 1443.10 = 4538.33 \text{ kip-ft.} \]

\[ S_{cf} = 16876.83(0.572) = 9653.55 \text{ kips-ft} > 4538.33 \text{ kip-ft.} \]

\[ M_{cr} = 4538.33 \text{ kip-ft.} \]

\[ 1.2 M_{cr} = 1.2(4538.33) = 5446.0 \text{ kip-ft.} \]

At midspan, factored moment required by Strength I load combination is:

\[ M_u = 7012.47 \text{ kip-ft.} \]

\[ 1.33M_u = 1.33(7012.47 \text{ K-ft.}) = 9326.59 \text{ kip-ft.} \]

Since \( 1.2M_{cr} < 1.33M_u \), the \( 1.2M_{cr} \) requirement controls.

\[ M_r = 9201.15 \text{ kip-ft} > 1.2M_{cr} = 5446.0 \text{ kip-ft.} \]

O.K.

The area and spacing of shear reinforcement must be determined at regular intervals along the entire length of the beam. In this design example, transverse shear design procedures are demonstrated below by determining these values at the critical section near the supports.

Transverse shear reinforcement is provided when:

\[ V_u < 0.5 \odot (V_c + V_p) \quad \text{[LRFD Art. 5.8.2.4-1]} \]

where

\[ V_u = \text{total factored shear force at the section, kips} \]

\[ V_c = \text{shear strength provided by concrete} \]

\[ V_p = \text{component of the effective prestressing force in the direction of the applied shear, Kips} \]

\( \odot = \text{resistance factor} = 0.90 \quad \text{[LRFD Art. 5.5.4.2.1]} \]
Critical Section

A.2.11.1

Critical section near the supports is the greater of:

\[ 0.5d_c \cot \theta \text{ or } d_v \]

where

\[ d_v = \text{effective shear depth} = \text{distance between resultants of tensile and compressive forces, } (d_{e-a/2}), \text{ but not less than the greater of } (0.9d_e) \text{ or } (0.72h) \]

\[ d_e = \text{the corresponding effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement} \]

\[ a = \text{depth of compression block} = 6.338 \text{ in. at midspan} \]

\[ h = \text{total height of section} = 62 \text{ in.} \]

\[ \theta = \text{angle of inclination of diagonal compressive stresses, assume } \theta = 23^\circ \text{ (slope of compression field)} \]

The shear design at any section depends on the angle of diagonal compressive stresses at the section. Shear design is an iterative process that begins with assuming a value for \( \theta \).

Since some of the strands are harped, the effective depth \( d_e \), varies from point to point. However \( d_e \) must be calculated at the critical section in shear which is not yet known; therefore, for the first iteration, \( d_e \) is calculated based on the center of gravity of the straight strand group at the end of the beam, \( y_{bend} \).

\[ d_e = h - y_{bend} = 62 - 5.33 = 56.67 \text{ in.} \]

\[ d_v = d_e - 0.5(a) = 56.67 - 0.5(6.338) = 53.50 \text{ in. (controls)} \]

\[ \geq 0.9d_e = 0.9(56.67) = 51.00 \text{ in. O.K.} \]

\[ \geq 0.72h = 0.72(62) = 44.64 \text{ in. O.K.} \]

Therefore \( d_e = 53.50 \text{ in.} \).

The critical section near the support is greater of:

\[ d_v = 53.50 \text{ in.} \]

and

\[ 0.5d_c \cot \theta = 0.5(53.50)(\cot 23^\circ) = 63.02 \text{ in. from face of the support (controls)} \]

Adding half the bearing width (3.25 in.) to critical section distance from face of the support to get the distance of the critical section from center line of bearing.

A.2.11.1.1

Angle of Diagonal Compressive Stresses

A.2.11.1.2

Effective Shear Depth

A.2.11.1.3

Calculation of critical section
AASHTO Type IV - LRFD Specifications

A.2.11.2 Contribution of Concrete to Nominal Shear Resistance

A.2.11.2.1 Strain in Flexural Tension Reinforcement

Critical section for shear is

\[ 63.02 + 3.25 = 66.27 \text{ in.} = 5.52 \text{ ft. from centerline of bearing} \]

\[ x/L = 5.52/108.583 = 0.05L \]

It is conservative not to refine the value of \( d_e \) based on the critical section 0.05L. The value if refined will have small difference.

The contribution of the concrete to the nominal shear resistance is:

\[ V_c = 0.0316 \beta \sqrt{f'_c b d_e} \quad [\text{LRFD Eq. 5.8.3.3-3}] \]

Calculate the strain in the reinforcement, \( \varepsilon_x \), on the flexural tension side. Assuming that the section contains at least the minimum transverse reinforcement as specified in LRFD Specifications Art. 5.8.2.5:

\[ \varepsilon_x = \frac{M_u + 0.5N_u + 0.5(V_u-V_p) \cot \theta - A_{p0} f_{po}}{2(E_A + E_{pA_p})} \leq 0.001 \quad [\text{LRFD Eq. 5.8.3.4.2-1}] \]

where

- \( V_u \) = applied factored shear force at the specified section, 0.05L
  \[ = 1.25(40.04 + 39.02 + 5.36) + 1.50(6.15) + 1.75(67.28 + 25.48) \]
  \[ = 277.08 \text{ kips} \]
- \( M_u \) = applied factored moment at the specified section, 0.05L > \( V_u d_e \)
  \[ = 1.25(233.54 + 227.56 + 31.29) + 1.50(35.84) + 1.75(291.58 + 116.33) \]
  \[ = 1383.09 \text{ Kip-ft.} > 277.08(53.5/12) = 1253.32 \text{ Kip-ft.} \quad \text{O.K.} \]
- \( N_u \) = applied factored normal force at the specified section, 0.05L = 0
- \( f_{po} \) = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked in difference in strain between the prestressing tendons and the surrounding concrete (ksi) For pretensioned members, LRFD Art. C5.8.3.4.2 indicates that \( f_{po} \) can be taken as the stress in strands when the concrete is cast around them, which is jacking stress \( f_{pj} \), or \( f_{pu} \).
  \[ = 0.75(270.0) = 202.5 \text{ ksi} \]
- \( V_p \) = component of the effective prestressing force in the direction of the applied shear = (Force per strand)(Number of harped strands)(sin\( \Psi \))
  \[ = \tan^{-1} \left( \frac{42.67}{49.4(12)} \right) = 0.072 \text{ rad.} \]
  \[ = 22.82(10) \sin (0.072) = 16.42 \text{ Kips} \]

Detailed Design Examples - 50
Values of $\beta$ and $\theta$

The values of $\beta$ and $\theta$ are determined using LRFD Table 5.8.3.4.2-1. Linear interpolation is allowed if the values lie between two rows.

\[
\varepsilon_x = \frac{1383.09(12)}{53.50} + 0.5(0.0) + 0.5(277.08 - 16.42)\cot 23^\circ - 42(0.153)202.5 \\
+ 0.5(0.0) + 0.5(277.08 - 16.42)\cot 23 - 42(0.153)202.5 \\
\leq 0.001
\]

\[
\varepsilon_x = \frac{1383.09(12)}{53.50} + 0.5(0.0) + 0.5(277.08 - 16.42)\cot 23 - 42(0.153)202.5 \\
\leq 0.001
\]

Since this value is negative, LRFD Eq. 5.8.3.4.2-3 should be used to calculate $\varepsilon_x$

\[
\varepsilon_x = \frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot \theta - A_{ps}f_{po}
\]

\[
\varepsilon_x = \frac{2}{2(E_cA_c + E_bA_b + E_pA_p)}
\]

where

- $A_c =$ area of the concrete (in.²) on the flexural tension side below $h/2$
  
  $A_c = 473$ in.²

- $E_c = (150)^{1.5}(33)\sqrt{5683.33} \frac{1}{1000} = 4570.38$ ksi

\[
\varepsilon_x = \frac{1383.09(12)}{53.50} + 0.5(0.0) + 0.5(277.08 - 16.42)\cot 23 - 42(0.153)202.5 \\
\leq 0.001
\]

$\varepsilon_x = -0.000146$

\[
\varepsilon_x = -0.000146
\]

\[
\varepsilon_x = \frac{V_u + \phi V_p}{\phi b_v d_v}
\]

where

- $\varepsilon_u =$ shear stress in concrete
- $\phi =$ resistance factor = 0.9
- $b_v =$ web width = 8 in.

\[
\varepsilon_u = \frac{277.08 - 0.9(16.42)}{0.9(8)(53.50)} = 0.681$ ksi

\[
\varepsilon_u / \Gamma_c = 0.681/5.683 = 0.12
\]

\[
\varepsilon_x = \frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot \theta - A_{ps}f_{po}
\]

\[
\varepsilon_x = \frac{2}{2(E_cA_c + E_bA_b + E_pA_p)}
\]

Detailed Design Examples - 51
Another iteration is made with $\theta = 20.65^\circ$ to arrive at the correct value of $\beta$ and $\theta$.

$$\theta = 20.65^\circ < 23^\circ \text{ (assumed)}$$

$$d_c = 56.67 \text{ in.}$$

$$d_v = 53.50 \text{ in.}$$

The critical section near the support is greater of:

$$d_v = 53.50 \text{ in.}$$

and

$$0.5d_v \cot \theta = 0.5(53.50)(\cot 20.65^\circ) = 71.0 \text{ in. from face of the support}$$

Adding half the bearing width (3.25 in.) to critical section distance from face of the support to get the distance of the critical section from center line of bearing.

Critical section for shear is

$$71.0 + 3.25 = 74.25 \text{ in.} = 6.19 \text{ ft. from centerline of bearing}$$

$$x/L = 6.19/108.583 = 0.057L$$

It is conservative not to refine the value of $d_v$ based on the critical section $0.057L$. The value if refined will have small difference.

Calculate the strain in the reinforcement, $\varepsilon_x$ on the flexural tension side.

Assuming that the section contains at least the minimum transverse reinforcement as specified in LRFD Specifications Art. 5.8.2.5:
Assuming the strain will be negative again, LRFD Eq. 5.8.3.4.2-3 should be used to calculate $\varepsilon_x$

$$\varepsilon_x = \frac{M_u + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{p0}f_{p0}}{2(E_cA_c + E_uA_u + E_pA_p)}$$

where

$V_u$ = applied factored shear force at the specified section, 0.057L

$$V_u = 1.25(39.49 + 38.48 + 5.29) + 1.50(6.06) + 1.75(66.81 + 25.15) = 274.10 \text{ kips}$$

$M_u$ = applied factored moment at the specified section, 0.058L > $V_u$d_v

$$M_u = 1.25(260.18 + 253.53 + 34.86) + 1.50(39.93) + 1.75(324.63 + 129.60) = 1540.50 \text{ Kip-ft.} > 274.10(53.5/12) = 1222.03 \text{ Kip-ft.} \text{ O.K.}$$

$N_u$ = applied factored normal force at the specified section, 0.057L = 0

$f_{p0} = 0.75(270.0) = 202.5 \text{ ksi}$

$V_p = 22.82(10) \sin (0.072) = 16.42 \text{ Kips}$

$A_c = 473 \text{ in.}^2$

$E_c = (150)^{1.5}(33)\sqrt{5683.33} = 4570.38 \text{ ksi}$

$$\varepsilon_x = \frac{1540.50(12) + 0.5(0.0) + 0.5(274.10 - 16.42)(\cot20.65^\circ) - 42(0.153)202.5}{2\left[4570.38(473) + 28000(0.0) + 28500(42)(0.153)\right]}$$

$$\varepsilon_x = -0.000131$$

$$\frac{V_u - \phi V_p}{\phi b_d c} = \frac{274.10 - 0.9(16.42)}{0.9(8)(53.50)} = 0.673 \text{ ksi} \quad \text{[LRFD Eq. 5.8.3.4.2-1]}$$

$$\frac{V_u}{f'c} = \frac{0.673}{5.683} = 0.118$$

<table>
<thead>
<tr>
<th>$\frac{V_u}{f'c}$</th>
<th>$\varepsilon_x \times 1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 0.200$</td>
<td>$-0.131$</td>
</tr>
<tr>
<td>$\leq 0.100$</td>
<td>$18.100$</td>
</tr>
<tr>
<td></td>
<td>$3.790$</td>
</tr>
<tr>
<td>$0.118$</td>
<td>$19.396$</td>
</tr>
<tr>
<td></td>
<td>$3.351$</td>
</tr>
<tr>
<td>$\leq 0.125$</td>
<td>$19.90$</td>
</tr>
<tr>
<td></td>
<td>$3.180$</td>
</tr>
</tbody>
</table>
\[ \theta = 20.83^0 \approx 20.65^0 \text{ (assumed)} \]

Therefore no further iteration is needed.

\[ \beta = 3.18 \]

where \( \beta \) = a factor indicating the ability of diagonally cracked concrete to transmit tension.

The nominal shear resisted by the concrete is:

\[ V_c = 0.0316\beta f' \sqrt{b.d.} \]

\[ V_c = 0.0316(3.18)\sqrt{5.683(8)(53.50)} = 102.53 \text{ kips} \]

Check if \( V_u > 0.5\phi (V_c + V_p) \)

\[ V_u = 274.10 \text{ kips} > 0.5(0.9)(102.53 + 16.42) = 53.53 \text{ kips} \]

Therefore, transverse shear reinforcement should be provided.

\[ V_s = \frac{V_u - V_c - V_p}{\phi} = \frac{274.10}{0.9} - \frac{102.53 - 16.42}{185.61} = 185.61 \text{ kips} \]

where

\[ A_v \cdot f_y \cdot d \cdot (\cot \theta + \cot \alpha) \sin \alpha \]

\[ V_s = \frac{s}{s} \]

\[ A_v = \text{area of shear reinforcement within a distance } s, \text{ in.}^2 \]

\[ s = \text{spacing of stirrups, in.} \]

\[ f_y = \text{yield strength of shear reinforcement, ksi} \]

\[ \alpha = \text{angle of inclination of transverse reinforcement to longitudinal axis} \]

\[ = 90^0 \text{ for vertical stirrups} \]

Therefore, area of shear reinforcement within a distance \( s \) is:

\[ A_v = \frac{sV_s}{f_y d (\cot \theta + \cot \alpha) \sin \alpha} \]
A.2.11.3.4

**Determine spacing of reinforcement**

Check for maximum spacing of transverse reinforcement: [LRFD Art. 5.8.2.7]

check if \( v_u < 0.125f'_c \) [LRFD Eq. 5.8.2.7-1]

or if \( v_u \geq 0.125f'_c \) [LRFD Eq. 5.8.2.7-2]

\[ 0.125f'_c = 0.125(5.683) = 0.71 \text{ ksi} \]

\[ v_u = 0.673 \text{ ksi} \]

Since \( v_u < 0.125f'_c \) therefore \( s \leq 24 \text{ in.} \) [LRFD Eq. 5.8.2.7-2]

\[ s \leq 0.8d_v = 0.8(53.50) = 42.80 \text{ in.} \]

Therefore maximum \( s = 24.0 \text{ in.} \) \( s \) provided O.K.

Use #4 bar double legs at 12 in. c/c, \( A_v = 2(0.20) = 0.40 \text{ in}^2/\text{ft} \)

\[ V_s = \frac{0.4(60)(53.50)(\cot 20.65^\circ)}{12} = 283.9 \text{ kips} \]


A.2.11.3.4

**Minimum Reinforcement requirement**

The area of transverse reinforcement should not be less than: [LRFD Art. 5.8.2.5]

\[ 0.0316\sqrt{f'_c} = 0.0316\sqrt{5.683} = 0.12 < A_v \text{ provided O.K.} \]

A.2.11.4

**Maximum Nominal Shear Resistance**

In order to assure that the concrete in the web of the beam will not crush prior to yield of the transverse reinforcement, the LRFD Specifications give an upper limit for \( V_n \) as follows:

\[ V_n = 0.25f'_c b_d + V_p \] [LRFD Eq. 5.8.3.3-2]

Comparing above equation with LRFD Eq. 5.8.3.3-1

\[ V_c + V_s \leq 0.25f'_c b_d \]

\[ 102.53 + 283.9 = 386.43 \text{ kips} \leq 0.25(5.683)(8)(53.50) = 608.08 \text{ kips} \]

This is a sample calculation for determining transverse reinforcement requirement at critical section and this procedure can be followed to find the transverse reinforcement requirement at increments along the length of the beam.
A.2.12
INTERFACE SHEAR TRANSFER

A.2.12.1
Factored Horizontal Shear

At the strength limit state, the horizontal shear at a section can be calculated as follows

$$V_h = \frac{V_u}{d_v}$$  \[LRFD Eq. C5.8.4.1-1\]

where

- $V_h$ = horizontal shear per unit length of the beam, kips
- $V_u$ = factored shear force at specified section due to superimposed loads, kips
- $d_v$ = the distance between resultants of tensile and compressive forces

$$d_v = (d_v - a/2), \text{ in}$$

The LRFD Specifications do not identify the location of the critical section. For convenience, it will be assumed here to be the same location as the critical section for vertical shear, at point 0.057L

Using load combination Strength I:

$$V_u = 1.25(5.29) + 1.50(6.06) + 1.75(66.81 + 25.15) = 176.63 \text{ kips}$$

$$d_v = 53.50 \text{ in}$$

Therefore applied factored horizontal shear is:

$$V_h = \frac{176.63}{53.50} = 3.30 \text{ kips/in.}$$

A.2.12.2
Required Nominal Resistance

Required $V_n = \frac{V_h}{\phi} = 3.30/0.9 = 3.67 \text{ kip/in}$

A.2.12.3
Required Interface Shear Reinforcement

The nominal shear resistance of the interface surface is:

$$V_n = cA_{cv} + \mu \left[A_{vf}f_y + P_c\right]$$ \[LRFD Eq. 5.8.4.1-1\]

where

- $c$ = cohesion factor \[LRFD Art. 5.8.4.2\]
- $\mu$ = friction factor \[LRFD Art. 5.8.4.2\]
- $A_{cv}$ = area of concrete engaged in shear transfer, in$^2$.
- $A_{vf}$ = area of shear reinforcement crossing the shear plane, in$^2$.
- $P_c$ = permanent net compressive force normal to the shear plane, kips
- $f_y$ = shear reinforcement yield strength, ksi
For concrete placed against clean, hardened concrete and free of laitance, but not an intentionally roughened surface:  
\[ c = 0.075 \text{ ksi} \]
\[ \mu = 0.6\lambda, \text{ where } \lambda = 1.0 \text{ for normal weight concrete, and therefore, } \mu = 0.6 \]
The actual contact width, \( b_v \), between the slab and the beam is 20 in.
\[ A_{cv} = (20 \text{ in.})(1 \text{ in}) = 20 \text{ in}^2 \]
The LRFD Eq. 5.8.4.1-1 can be solved for \( A_{cf} \) as follows:
\[ 3.67 = (0.075)(20) + 0.6(A_{cf}(60) + 0) \]
Solving for \( A_{cf} = 0.06 \text{ in}^2/\text{in} \) or 0.72 in\(^2\)/ft
2 - #4 double-leg bar per ft are provided.
Area of steel provided = 2 (0.40) = 0.80 in\(^2\)/ft
Provide 2 legged #4 bars at 6 in. c/c
The web reinforcement shall be provided at 6 in. c/c which can be extended into the cast in place slab to account for the interface shear requirement.

Minimum \( A_{cf} \geq (0.05b_v)/f_y \)  
[LRFD Eq. 5.8.4.1-4]
where \( b_v \) = width of the interface
\[ A_{cf} = 0.80 \text{ in}^2/\text{ft} > [0.05(20)/60](12 \text{ in./ft}) = 0.2 \text{ in}^2/\text{ft}. \text{ O.K.} \]
\[ V_n \text{ provided} = 0.075(20) + 0.6 \left( \frac{0.80}{12} (60) + 0 \right) = 3.9 \text{ kips/in.} \]
\[ 0.2f'cA_{cv} = 0.2(4.0)(20) = 16 \text{ kips/n.} \]
\[ 0.8A_{cv} = 0.8(20) = 16 \text{ kips/in.} \]
Since provided \( V_n \leq 0.2 f'cA_{cv} \text{ O.K.} \)  
[LRFD Eq. 5.8.4.1-2]
\[ \leq 0.8A_{cv} \text{ O.K.} \]  
[LRFD Eq. 5.8.4.1-3]

Longitudinal reinforcement should be proportioned so that at each section the following equation is satisfied
\[ A_{s}f_y + A_{ps}f_{ps} \geq \frac{M_u}{d_{c,\varphi}} + 0.5\frac{N_u}{\varphi} + \left( \frac{V_u}{\varphi} - 0.5V_s - V_p \right)\cot\theta \]  
[LRFD Eq. 5.8.3.5-1]
where
\[ A_s = \text{area of non prestressed tension reinforcement, in}^2 \]

Detailed Design Examples - 57
AASHTO Type IV - LRFD Specifications

\[ f_y = \text{specified minimum yield strength of reinforcing bars, ksi} \]
\[ A_{ps} = \text{area of prestressing steel at the tension side of the section, in.}^2 \]
\[ f_{ps} = \text{average stress in prestressing steel at the time for which the nominal resistance is required, ksi} \]
\[ M_u = \text{factored moment at the section corresponding to the factored shear force, kip-ft.} \]
\[ N_u = \text{applied factored axial force, kips} \]
\[ V_u = \text{factored shear force at the section, kips} \]
\[ V_s = \text{shear resistance provided by shear reinforcement, kips} \]
\[ V_p = \text{component in the direction of the applied shear of the effective prestressing force, kips} \]
\[ d_v = \text{effective shear depth, in.} \]
\[ \phi = \text{resistance factor as appropriate for moment, shear and axial resistance.} \]

Therefore different \( \phi \) factors will be used for terms in Eq. 5.8.3.5-1, depending on the type of action being considered [LRFD Art. 5.5.4.2]

\[ \theta = \text{angle of inclination of diagonal compressive stresses.} \]

\[ \text{[LRFD Art. 5.8.3.5]} \]

Width of bearing = 6.5 in.
Distance of section = 6.5/2 = 3.25 in. = 0.271 ft.
Shear forces and bending moment are calculated at this section
\[ V_u = 1.25(44.35 + 43.22 + 5.94) + 1.50(6.81) + 1.75(71.05 + 28.14) \]
\[ = 300.69 \text{ kips.} \]
\[ M_u = 1.25(12.04 + 11.73 + 1.61) + 1.50(1.85) + 1.75(15.11 + 6.00) \]
\[ = 71.44 \text{ Kip-ft.} \]
\[ \frac{M_u}{d_v \phi} + 0.5 \frac{N_u}{\phi} + \left( \frac{V_u}{\phi} - 0.5V_s - V_p \right) \cot \theta \]
\[ = 71.44(12) + 0 + \left( \frac{300.69}{0.9} - 0.5(283.9) - 16.42 \right) \cot 20.65^\circ = 484.09 \text{ kips} \]

The crack plane crosses the centroid of the 42 straight strands at a distance of 6 + 5.33 \( \cot 20.65^\circ = 20.14 \) in. from the end of the beam. Since the transfer length is 30 in. the available prestress from 42 straight strands is a fraction of the effective prestress, \( f_{ps} \), in these strands. The 10 harped strands do not contribute the tensile capacity since they are not on the flexural tension side of the member.
Therefore available prestress force is:

\[ A_s f_y + A_{ps} f_{ps} = 0 + 42(0.153) \left( \frac{149.18 \times 20.33}{30} \right) = 649.63 \text{ Kips} \]

\[ A_s f_y + A_{ps} f_{ps} = 649.63 \text{ kips} > p \]

\[ v \]

Therefore additional longitudinal reinforcement is not required.

**A.2.14 PRETENSIONED ANCHORAGE ZONE**

**A.2.14.1 Minimum Vertical Reinforcement**

Design of the anchorage zone reinforcement is computed using the force in the strands just prior to transfer:

Force in the strands at transfer = \( F_{pi} = 52 \times 0.153 \times 202.5 = 1611.09 \text{ kips} \)

The bursting resistance, \( P_r \), should not be less than 4% of \( F_{pi} \)

\[ P_r = f_s A_s \geq 0.04 F_{pi} = 0.04(1611.09) = 64.44 \text{ kips} \]

where

- \( A_s \) = total area of vertical reinforcement located within a distance of \( h/4 \) from the end of the beam, in\(^2\).
- \( f_s \) = stress in steel not exceeding 20 ksi.

Solving for required area of steel \( A_s = 64.44/20 = 3.22 \text{ in}^2 \)

At least 3.22 in\(^2\) of vertical transverse reinforcement should be provided within a distance of \( (h/4 = 62 / 4 = 15.5 \text{ in}) \) from the end of the beam.

Use 6 - #5 double leg bars at 2.0 in. spacing starting at 2 in. from the end of the beam.

The provided \( A_s = 6(2)0.31 = 3.72 \text{ in}^2 > 3.22 \text{ in}^2 \) O.K.

**A.2.14.2 Confinement Reinforcement**

For a distance of \( 1.5d = 1.5(54) = 81 \text{ in} \) from the end of the beam, reinforcement is placed to confine the prestressing steel in the bottom flange. The reinforcement shall not be less than #3 deformed bars with spacing not exceeding 6 in. The reinforcement should be of shape which will confine the strands.
A.2.15
DEFLECTION AND CAMBER
A.2.15.1
Maximum Camber calculations using Hyperbolic Functions Method

TxDOT’s prestressed bridge design software, PSTRS 14 uses the Hyperbolic Functions Method proposed by Sinno Rauf, and Howard L Furr (1970) for the calculation of maximum camber. This design example illustrates the PSTRS 14 methodology for calculation of maximum camber.

Step 1: Total Prestress after release

\[
P = \frac{P_{si}}{1 + \frac{p\varepsilon_c A_n}{I}} + \frac{M_n\varepsilon_c A_n}{I(1 + \frac{p\varepsilon_c A_n}{I})}
\]  

where

- \(P_{si}\) = total prestressing force = 1461.28 Kips
- \(I\) = moment of inertia of non-composite section = 260403 in.\(^4\)
- \(\varepsilon_c\) = eccentricity of pretensioning force at the midspan = 19.29 in.
- \(M_n\) = Moment due to self weight of the beam at midspan = 1209.98 K-ft.
- \(A_s\) = Area of strands = number of strands (area of each strand) = 52(0.153) = 7.96 in.\(^2\)
- \(p = \frac{A_s}{A}\)
  where
  - \(A\) = Area of cross section of beam = 788.4 in.\(^2\)
  - \(p = 7.96/788.4 = 0.0101\)
- \(E_c\) = modulus of elasticity of the beam concrete at release, ksi
  - \(E_c = 33(w_c)^{3/2} f_{c'}^{1.5} \) \[STD Eq. 9-8\]
  - \(= 33(150)^{3/2} \sqrt{5683.33} \frac{1}{1000} = 4570.38\) ksi
- \(E_p\) = Modulus of elasticity of prestressing strands = 28500 ksi
  - \(n = \frac{E_p}{E_c} = 28500/4570.38 = 6.236\)
  - \(\left(1 + p\varepsilon_c A_n/I\right) = 1 + 0.0101(6.236) + \frac{(19.29)^2(7.96)(6.236)}{260403} = 1.134\)

\[
P = \frac{P_{si}}{1 + \frac{\varepsilon_c^2 A_n}{I}} + \frac{M_n\varepsilon_c A_n}{I(1 + \frac{\varepsilon_c^2 A_n}{I})}
\]  

\[
= \frac{1461.28}{1.134} + \frac{(1209.98)(12 \text{ in./ft.})(19.29)(7.96)(6.236)}{260403(1.134)}
\]
Concrete Stress at steel level immediately after transfer

\[ f'_c = P \left( \frac{1}{A} + \frac{e_c^2}{I} \right) - f'_c \]

where

\[ f'_c = \text{Concrete stress at steel level due to dead loads} \]
\[ = \frac{M_c e_c}{I} = \frac{(1209.98)(12 \text{ in./ft.})(19.29)}{260403} = 1.0756 \text{ ksi} \]

\[ f'_{cl} = 1335.69 \left( \frac{1}{788.4} + \frac{19.29^2}{260403} \right) - 1.0756 = 2.527 \text{ ksi} \]

Step 2: Ultimate time-dependent strain at steel level

\[ \varepsilon_{s1} = \varepsilon_{cr} f'_{cl} + \frac{\varepsilon_s}{\varepsilon_{sh}} \]

where

\[ \varepsilon_{cr} = \text{ultimate unit creep strain} = 0.00034 \text{ in./in. (this value is prescribed by Sinno et. al. (1970))} \]
\[ \varepsilon_{sh} = \text{ultimate unit creep strain} = 0.000175 \text{ in./in. (this value is prescribed by Sinno et. al. (1970))} \]

\[ \varepsilon_{s1} = 0.00034(2.527) + 0.000175 = 0.001034 \text{ in./in.} \]

Step 3: Adjustment of total strain in step 2

\[ \varepsilon_{s2} = \varepsilon_{s1} - \varepsilon_{s1} E_p A \left( \frac{1}{A} + \frac{e_c^2}{I} \right) \]

\[ = 0.001034 - 0.001034(28500) \frac{7.96}{4570.38} \left( \frac{1}{788.4} + \frac{19.29^2}{260403} \right) = 0.000896 \text{ in./in.} \]

Step 4: Change in concrete stress at steel level

\[ \Delta f'_c = \varepsilon_{s2} E_p A \left( \frac{1}{A} + \frac{e_c^2}{I} \right) = 0.000896(28500)(7.96) \left( \frac{1}{788.4} + \frac{19.29^2}{260403} \right) \]

\[ \Delta f'_c = 0.548 \text{ ksi} \]
Step 5: Correction of the total strain from step 2

\[ \varepsilon_{s4}^{c} = \varepsilon_{cr}^\infty \left( \varepsilon_{c1}^s - \frac{\Delta f_{c1}^s}{2} \right) + \varepsilon_{sh}^\infty \]

\[ \varepsilon_{s4}^c = 0.00034 \left( 2.527 - \frac{0.548}{2} \right) + 0.000175 = 0.000941 \text{ in./in.} \]

Step 6: Adjustment in total strain from step 5

\[ \varepsilon_{s5}^{c} = \varepsilon_{s4}^{c} - \varepsilon_{s4}^{c} E_{c} \left( \frac{A_{s}}{A} + \frac{e_{c}^{2}}{I} \right) \]

\[ = 0.000941 - 0.000941(28500) \frac{7.96}{4570.38} \left( \frac{1}{788.4} + \frac{19.29^2}{260403} \right) = 0.000815 \text{ in./in} \]

Step 7: Change in concrete stress at steel level

\[ \Delta f_{c1} = \varepsilon_{s5}^{c} E_{p} A_{s} \left( \frac{1}{A} + \frac{e_{c}^{2}}{I} \right) = 0.000815(28500)(7.96) \left( \frac{1}{788.4} + \frac{19.29^2}{260403} \right) \]

\[ \Delta f_{c1} = 0.499 \text{ ksi} \]

Step 8: Correction of the total strain from step 5

\[ \varepsilon_{s6}^{c} = \varepsilon_{cr}^\infty \left( \varepsilon_{c1}^s - \frac{\Delta f_{c1}^s}{2} \right) + \varepsilon_{sh}^\infty \]

\[ \varepsilon_{s6}^c = 0.00034 \left( 2.527 - \frac{0.499}{2} \right) + 0.000175 = 0.000949 \text{ in./in.} \]

Step 9: Adjustment in total strain from step 8

\[ \varepsilon_{s7}^{c} = \varepsilon_{s6}^{c} - \varepsilon_{s6}^{c} E_{p} \left( \frac{A_{s}}{A} + \frac{e_{c}^{2}}{I} \right) \]

\[ = 0.000949 - 0.000949(28500) \frac{7.96}{4570.38} \left( \frac{1}{788.4} + \frac{19.29^2}{260403} \right) = 0.000822 \text{ in./in} \]

Step 10: Computation of initial prestress loss

\[ PL_i = \frac{\Psi - P}{\Psi} = \frac{1461.28 - 1335.69}{1461.28} = 0.0859 \]
Step 11: Computation of Final Prestress loss

\[ PL^\infty = \frac{\epsilon^P_{c_e}EPa_s}{P_{si}} = \frac{0.000822(28500)(7.96)}{1461.28} = 0.128 \]

Total Prestress loss

\[ PL = PL_i + PL^\infty = 100(0.0859 + 0.128) = 21.39\% \]

Step 12: Initial deflection due to dead load

\[ C_{DL} = \frac{5wL^4}{384EI} \]

where

- \( w = \) weight of beam = 0.821 kips/ft.
- \( L = \) span length = 108.583 ft.

\[ C_{DL} = \frac{5(0.821)}{12 \text{ in./ft.}}[\frac{(108.583)(12 \text{ in./ft.})^4}{384(4570.38)(260403)}] = 2.158 \text{ in.} \]

Step 13: Initial Camber due to prestress

\[ C_{pi} = [0.5P(e_c)(0.5L)^2 + 0.5P(e_c - e_s)(0.67)(HD)^2 + 0.5P(e_c - e_s)(HD_{dis})(0.5L + HD)]/(Ec)(I) \]

where

- \( HD = \) Hold down distance from support = 48.87 ft. = 586.44 in.
- \( HD_{dis} = \) Hold down distance from center = 0.5(108.583) – 48.87 = 5.422 ft. = 65.06 in.
- \( e_c = \) Eccentricity at support = 11.21 in.
- \( L = \) span length = 108.583 ft. = 1303 in.

\[ C_{pi} = [0.5(1335.69)(11.21)[(0.5)(1303)]^2 + 0.5(1335.69)(19.29 - 11.21)(0.67)(586.44)^2 + 0.5(1335.69)(19.29 - 11.21)(65.06)(0.5)(1303) + 586.44)]/[4570.38(260403)] \]

\[ C_{pi} = [3.178*10^9 + 1.243*10^9 + 0.4346*10^9]/1.19*10^9 = 4.08 \text{ in.} \]

Step 14: Initial Camber

\[ C_i = C_{pi} - C_{DL} = 4.080 - 2.158 = 1.922 \text{ in.} \]

Step 15: Ultimate Time Dependent Camber

Ultimate strain \( \epsilon_s^* = \frac{f^*_{c_e}}{E_c} = 2.527/4570.37 = 0.000553 \text{ in./in.} \)
Deflection due to slab weight

\[ \Delta_{\text{slab}} = \frac{5w_s L^4}{384E_c I} \]

where

- \( w_s \) = slab weight = 0.8 Kips/ft.
- \( E_c \) = modulus of elasticity of beam concrete at service
  
  \[ = (150)^{1.5}(33)\sqrt{5683.33} \frac{1}{1000} = 4570.38 \text{ ksi} \]
- \( I \) = moment of inertia of non-composite section = 260403 in.\(^4\)
- \( L \) = span length = 108.583 ft.

\[ \Delta_{\text{slab}} = \frac{5(0.8/12)((108.583)(12))^4}{384(4570.38)(260403)} = 2.10 \text{ in.} = 0.175 \text{ ft.} \]

Deflection at quarter span due to slab weight

\[ \Delta_{\text{slab}} = \frac{57w_s L^4}{6144E_c I} \]

\[ \Delta_{\text{slab}} = \frac{57(0.8/12)((108.583)(12))^4}{6144(4570.38)(260403)} = 1.498 \text{ in.} = 0.1248 \text{ ft.} \]

Deflection due to barrier weight at midspan

\[ \Delta_{\text{barr}} = \frac{5w_{\text{barr}} L^4}{384E_c I} \]

where

- \( w_{\text{barr}} \) = barrier weight = 0.11 Kips/ft.

Ultimate camber

\[ C_i = C_i(1 - PL) \]

\[ = 1.922(1 - 0.128) \]

\[ C_i = 4.023 \text{ in.} = 0.335 \text{ ft.} \]
Deflection at quarter span due to barrier weight

\[ \Delta_{\text{barr}1} = \frac{5(0.11/12)[(108.583)(12)]^4}{384(4570.38)(694599.5)} = 0.108 \text{ in.} = 0.009 \text{ ft.} \]

Deflection at quarter span due to barrier weight

\[ \Delta_{\text{barr}2} = \frac{57(0.11/12)[(108.583)(12)]^4}{6144(4570.38)(694599.5)} = 0.0772 \text{ in.} = 0.0064 \text{ ft.} \]

Deflection due to wearing surface weight at midspan

\[ \Delta_{\text{ws}1} = \frac{5w_{\text{ws}}L^4}{384E_I_c} \]

where

\( w_{\text{ws}} = \) wearing surface weight = 0.1259 Kips/ft.

\[ \Delta_{\text{ws}1} = \frac{5(0.1259/12)[(108.583)(12)]^4}{384(4570.38)(694599.5)} = 0.124 \text{ in.} = 0.01 \text{ ft.} \]

Deflection at quarter span due to barrier weight

\[ \Delta_{\text{ws}2} = \frac{57w_{\text{ws}}L^4}{6144E_I_c} \]

\[ \Delta_{\text{ws}2} = \frac{57(0.1259/12)[(108.583)(12)]^4}{6144(4570.38)(694599.5)} = 0.0884 \text{ in.} = 0.0074 \text{ ft.} \]
A1.16 COMPARISON OF RESULTS OBTAINED FROM STANDARD DETAILED DESIGN AND PSTRS 14 WITH THAT OF THE LRFD DETAILED DESIGN

The prestressed beam design program, PSTRS14 is used by TxDOT to design the bridges. PSTRS14 program was run with same parameters as used in this detailed design and the results are compared in table A.1.16.1. Also the results from detailed design using AASHTO Standard Specifications are compared.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LRFD Detailed Design Results</th>
<th>PSTRS 14 Results</th>
<th>Percent Difference</th>
<th>Standard Detailed Design Results</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Live Load Distribution factor</td>
<td>0.639</td>
<td>0.727</td>
<td>-13.77 %</td>
<td>0.727</td>
<td>-13.77 %</td>
</tr>
<tr>
<td>Moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear</td>
<td>0.814</td>
<td>0.727</td>
<td>10.69 %</td>
<td>0.727</td>
<td>10.69 %</td>
</tr>
<tr>
<td>Initial Prestress loss</td>
<td>9.30 %</td>
<td>8.93%</td>
<td>3.98 %</td>
<td>8.94%</td>
<td>3.87 %</td>
</tr>
<tr>
<td>Final Prestress Loss</td>
<td>26.33 %</td>
<td>25.23%</td>
<td>4.18 %</td>
<td>25.25%</td>
<td>4.10 %</td>
</tr>
</tbody>
</table>

| Beam Stresses at release           |                              |                  |                    |                                  |                    |
| At beam end:                       |                              |                  |                    |                                  |                    |
| at top fiber                       | 13 psi                       | 35 psi           | -169.23 %          | 35 psi                           | -169.23 %          |
| at bottom fiber                    | 3410 psi                     | 3274 psi         | 3.99 %             | 3274 psi                         | 3.99 %             |
| At transfer length section:        |                              |                  |                    |                                  |                    |
| at top fiber                       | 94 psi                       | Not Calculated   | -                   | 103 psi                          | -9.57 %            |
| at bottom fiber                    | 3342 psi                     | Not Calculated   | -                   | 3216 psi                         | 3.77 %             |
| At Hold Down:                      |                              |                  |                    |                                  |                    |
| at top fiber                       | 334 psi                      | 319 psi          | 4.49 %             | 354 psi                          | -5.99 %            |
| at bottom fiber                    | 3138 psi                     | 3034 psi         | 3.31 %             | 3005 psi                         | 4.24 %             |
| At Midspan:                        |                              |                  |                    |                                  |                    |
| at top fiber                       | 351 psi                      | 335 psi          | 4.56 %             | 368 psi                          | -4.84 %            |
| at bottom fiber                    | 3124 psi                     | 3020 psi         | 3.33 %             | 2991 psi                         | 4.26 %             |

| Beam Stresses at service loads     |                              |                  |                    |                                  |                    |
| At beam end:                       |                              |                  |                    |                                  |                    |
| at top fiber                       | Not Calculated               | 29 psi           | -                   | Not Calculated                   | -                  |
| at bottom fiber                    | Not Calculated               | 2688 psi         | -                   | Not Calculated                   | -                  |
| At Midspan under total loads:      |                              |                  |                    |                                  |                    |
| at top fiber                       | 2679 psi                     | 2562 psi         | 4.37 %             | 2562 psi                         | 4.37 %             |
| at bottom fiber                    | -444 psi                     | -413 psi         | 6.98 %             | -411 psi                         | 7.43 %             |
| Slab Top Stresses under total loads|                              |                  |                    |                                  |                    |
| Release Concrete Strength required | 5683.33 psi                  | 5457 psi         | 3.98 %             | 5457 psi                         | 3.98 %             |
| Final Concrete Strength required   | 5683.33 psi                  | 5585 psi         | 1.73 %             | 5585 psi                         | 1.73 %             |

<p>| Total No. of Strands              | 52                           | 50               | 3.85 %             | 50                               | 3.85 %             |
| No. of Draped Strands             | 10                           | 10               | 0.00 %             | 10                               | 0.00 %             |
| Ultimate Moment Required          | 7012.47 K-ft.                | 6769 K-ft.       | 3.47 %             | 6767.27 K-ft.                    | 3.50 %             |
| Ultimate Moment Provided          | 9201.15 K-ft.                | 8805 K-ft.       | 4.31 %             | 8936.56 K-ft.                    | 2.88 %             |
| Shear Reinforcement               | #4 bars at 6 in. c/c         | #3 bars at 12 in. c/c | #3 bars at 12 in. c/c | #3 bars at 12 in. c/c | #3 bars at 12 in. c/c |
| Area of shear Reinforcement       | 0.40 in.²/ft.               | 0.11 in.²/ft.    | 72.50 %            | 0.11 in.²/ft.                    | 72.50 %            |</p>
<table>
<thead>
<tr>
<th></th>
<th>Maximum Camber</th>
<th>0.335 ft.</th>
<th>0.318 ft.</th>
<th>5.35 %</th>
<th>0.320 ft.</th>
<th>5.33 %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deflections</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slab Weight: At midspan</td>
<td>-0.175 ft.</td>
<td>-0.1767 ft.</td>
<td>-0.97 %</td>
<td>-0.177 ft.</td>
<td>-1.14 %</td>
<td></td>
</tr>
<tr>
<td>At Quarter span</td>
<td>-0.1248 ft.</td>
<td>-0.1259 ft.</td>
<td>-0.88 %</td>
<td>-0.1259 ft.</td>
<td>-0.88 %</td>
<td></td>
</tr>
<tr>
<td>Barrier Weight: At midspan</td>
<td>-0.0090 ft.</td>
<td>-0.0091 ft.</td>
<td>-1.11 %</td>
<td>-0.0091 ft.</td>
<td>-1.11 %</td>
<td></td>
</tr>
<tr>
<td>At quarter span</td>
<td>-0.0064 ft.</td>
<td>-0.0065 ft.</td>
<td>-1.56 %</td>
<td>-0.0065 ft.</td>
<td>-1.56 %</td>
<td></td>
</tr>
<tr>
<td>Wearing surface weight: At midspan</td>
<td>-0.01 ft.</td>
<td>-0.0104 ft.</td>
<td>-4.00 %</td>
<td>-0.01 ft.</td>
<td>0.00 %</td>
<td></td>
</tr>
<tr>
<td>At quarter span</td>
<td>-0.0074 ft.</td>
<td>-0.0074 ft.</td>
<td>0.00 %</td>
<td>-0.0074 ft.</td>
<td>0.00 %</td>
<td></td>
</tr>
</tbody>
</table>